Combining optimizer and metamodelling for railcar structural optimization

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Abstract: Stress constraint is a hard issue for structural topology optimization, especially for large-scale structures, e.g. railcars. Another technique is proposed to combine a sizing optimizer with metamodelling for topology optimization. At the lower level, for each topology design sampled within the topology design space, a sizing optimizer finds feasible and optimal solutions in terms of sizing variables (plate thickness in continuum structures). All performance constraints such as stress, displacement, and stability, are handled only at this level. At the upper level, a metamodel is built to fit all the optimal solutions found at the lower level and is optimized for topology design. The only constraints left at the upper level are topological constraints and topological variable bounds. Only the objective function (e.g. weight) versus topological variables, and not the constraints, is approximated. The number of topology design variables is much smaller than those used in many other topology optimization approaches. Thus it may be able to handle large-scale structural systems. It was applied to two boxcar design projects, resulting in 18 per cent and 36 per cent weight savings and significant reductions in manufacturing cost and total cost.

Keywords: railcar optimization, stress constraint, metamodelling, structural systems, topology optimization

1 INTRODUCTION

There have been extensive research and development in structural topology optimization in recent decades [**1–10**]. However, the practical techniques to address real design needs such as minimizing weight with stress and other local constraints are yet to be fully developed, especially for large-scale structural systems [**1**, **11**]. Until the current technologies become more mature, design engineers continue to search or test the tools that can improve real-world designs. Applying response surface methods or metamodel optimization is one of the tools.

The typical approach has been to approximate the constraints as well as the objective function at the same time. This may be problematic for many structures.

For shape optimization, Wang *et al.* [**12**] proposed a two-level decomposition method with sizing variables handled at the first level and shape variables at the second level, but all the constraints (above some critical values) are handled at both levels. This too may be problematic for topology optimization of real-world structures.

A two-level or hierarchical, interactive, and metamodel-based optimization (HIMO) approach that does not require the topology optimization to be combined with the lower-level problem of satisfying the major performance constraints such as strength is proposed here. There are usually large numbers of such local constraints, even with small structures, resulting in large-scale problems that are expensive to solve. Why are existing techniques for structural analysis and sizing optimization not used in order to separate this 'lower-level' analysis and optimization from the topology optimization? Using such a hierarchical approach, at the upper level, only the topology constraints need to be satisfied, making the topology optimization much easier.

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In many topology optimization methods, a large number of topology design variables are involved, making it very difficult or expensive to optimize large-scale structural systems consisting of many subsystems or elements. Essentially, many methods currently in use focus on optimizing components, rather than the overall structural system. In order to solve large-scale problems, it is necessary to reduce the number of topology design variables.

Although the theory of structural topology optimization has enjoyed considerable development, many challenges remain (see, for example, references [**1**] and [**13**]). For instance, some major topology optimization methods ask the users to specify the volume portion of the final structure versus the whole volume of the reference or design domain before starting optimization. Further, the volume portion serves as the only constraint, but, in many real cases, the designers do not know nor care about the portion before optimization and stress is in fact the most important constraint.

There has been some research on optimizing railcars, (see, for example, reference [**14**] for a study on passenger cars). This paper focuses on optimizing structures for railroad freight cars.

2 A NEW APPROACH: HIMO

This study proposes a systematic approach that can be practical and effective for layout optimization of structural systems. It combines a sizing optimizer and metamodel optimization through a sequence of computer experiments.

Unlike the simple approximation shown in Fig. 1, HIMO goes through many thickness options to find optimal thickness for every option of topology design as shown in Fig. 2, at the lower level before building the metamodel. In both figures, X is the topology variable, supposing only one, and Y is the objective function, e.g. weight.

In Fig. 2, for every topology design, among all the thickness options, some are feasible (indicated as open circles), but the others are not (indicated as croses). Among all the feasible options, there is one that has the minimum objective function value. That is the optimal point for this topology design.

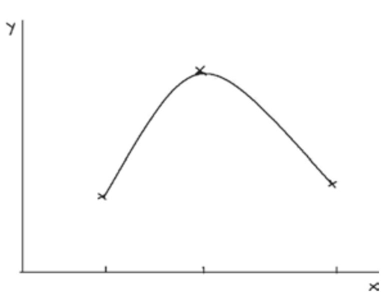


Fig. 1 Simple approximation

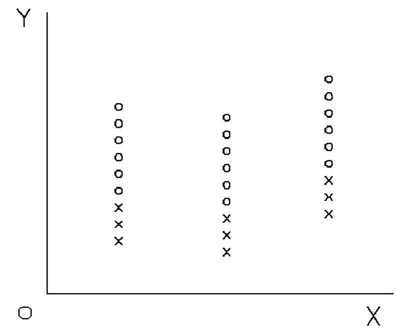


Fig. 2 HIMO: low level

At the upper level (Fig. 3) an optimal curve or surface is built to fit all the optimal points for every topological variable value. The topology optimization is to optimize the optimal surface to find the best topology design with the lowest objective function value.

The procedure of HIMO is outlined below.

Step 1. The statistical techniques for computerized experimental design are used to generate the sampling points in the design space, representing different configurations or layout options of the structural systems. Latin hypercube design and Latin hypercube plus minimizing the sum of the inverse of the square distances between points and/or minimizing the centred discrepancy were used in this study.

Step 2. Finite element analyses (FEAs) are run for every sampled design. Adjustment is made to reach feasible designs in terms of the major performance constraints such as stress, displacement, stability, as well as bound limits for sizing or property variables. At the same time or later, sizing-optimal or optimum thickness (in continuum cases) designs can be reached by sizing optimization. Each resulting feasible and optimal design corresponds to one of the configuration or layout design points determined in step 1.

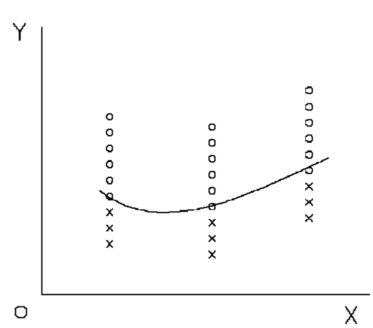


Fig. 3 HIMO: both levels

Step 3. A metamodel representing the objective function, e.g. the structural weight, versus the topology design variables, is then built to fit the optimal solutions of the computer experiments. Every point on this response surface is assumed to be the feasible and optimal design in terms of the major performance constraints and thickness optimization, for every topology design. Kriging models were used in this study.

Step 4. The upper level optimization is done to find the best point on the response surface, corresponding to the optimal layout design, satisfying the bound constraints and topology constraints of the topology variables. Sequential quadratic programming (SQP) or genetic algorithms followed by SQP were used in this study.

Step 5. An FEA is run again to work on the topologically optimal solution just obtained to obtain the final result feasible and optimal in terms of both sizing and topology.

When the topology design space is large, sequential sampling can be used on the basis of the result of a previous cycle of the five steps, as is shown in section 4. The interaction between the designer and the computer forms an interactive approach.

To employ HIMO successfully, the metamodelling strategies addressing sampling, model selection and building, and optimization are critical. Sampling or experimental design is the most important, since effective and efficient sampling can provide uniform and representative coverage of the space investigated. Sometimes, even without further work on modelling and optimization, the sampling tests can direct the designers towards much better designs that might be sufficient for the task at hand. It may be more important to have enough samples than better designs for many cases, according to the present authors' simulation tests on approximating some test functions for prediction accuracy. The details will be published in another paper.

In Latin hypercube sampling (LHS), the *j*th component of the *i*th sampled point is

$$X_{ij} = \frac{\Pi_{ij} - U_{ij}}{n} \tag{1}$$

where the Π_{ij} is the *j*th element of the *i*th independent uniform random permutations of the integers 1 to n (n is the number of samples), and U_{ij} is the *j*th element of the *i*th independent $U[0, 1]$ (uniform distribution between 0 and 1) random variables independent of the Π_j [**15–17**]. LHS was introduced by McKay *et al.* [**17**] in 'the first paper on computer experiments'. The stratification in sampling usually increases the accuracy of the approximation of the models, when compared with simple random sampling. An LHS function is included in the kriging program mentioned below [**18**].

Leary *et al.* [**19**] proposed a distance-related metric C as an alternative to the metric used by Morris and Mitchell [**20**]. C is given by

$$C = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \frac{1}{d_{ij}^2} \tag{2}$$

where n is the number of sampled points and d_{ij} is the Euclidean distance between points i and j. Minimization of this quantity is sought.

Fang *et al.* [**21**] recommended Hickernell's centred analytical L_2 discrepancy CL_2 (n is the number of samples, s the number of dimensions, and P_n a set of n points) according to

$$\begin{aligned}[CL_2(P_n)]^2 = {} & \left(\frac{13}{12}\right)^2 - \frac{2}{n}\sum_{k=1}^{n}\prod_{j=1}^{s} \\ & \times \left(1 + \frac{1}{2}|x_{kj} - 0.5| - \frac{1}{2}|x_{kj} - 0.5|^2\right) \\ & + \frac{1}{n^2}\sum_{k=1}^{n}\sum_{j=1}^{n}\prod_{i=1}^{s}\left(1 + \frac{1}{2}|x_{ki} - 0.5| \right. \\ & \left. + \frac{1}{2}|x_{ji} - 0.5| - \frac{1}{2}|x_{ki} - x_{ji}|\right)\end{aligned} \tag{3}$$

Some relatively new approximation models such as kriging models are very powerful and extremely flexible, resulting in much more accurate global approximation than traditional low-order polynomials. Kriging models have the form

$$y(x) = f(x) + Z(x) \tag{4}$$

where $y(x)$ is the unknown function of interest. $f(x)$ is a known polynomial function of x, a global regression model, often a constant term. $Z(x)$ is the correlation model, often the realization of a Gaussian random process with mean zero, variance σ^2, and non-zero covariance (see reference [**22**] and [**23**] for details). An excellent kriging program can be downloaded from reference [**18**].

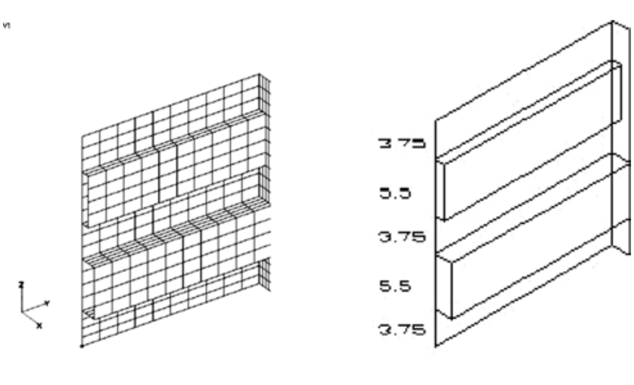
Usually bumpy metamodels with high non-linearity make global optimization tasks very difficult for mathematical programming approaches. The hybrid approaches such as genetic algorithms followed by SQP are often effective to address the global optimization. With the effective ways for sampling, modelling, and optimization, HIMO provides a systematic way for exploring the design space, which can be effective and efficient. More research is going on and many details regarding metamodelling and optimization strategies for HIMO will be discussed in new papers.

3 PILOT TESTS

As an initial trial to see the applicability and potential of HIMO, pilot tests were carried out on part of a real structure. This simple assembly is the bottom section of a boxcar end structure, consisting of an end sheet reinforced by two channels and a corner post. For FEA, only half is modelled because of the symmetry of the structure, loading, and constraints. The FEA model (for plot only) is shown in Fig. 4. The border is pinned so that all the translation movement is restricted, except the symmetric centre where the symmetric constraint is applied to simulate the other half. For visibility, only two variables are selected: the depths of the two channels. The objective function is the weight of the assembly. The layout question is what combination of the two channel depth results in the lowest weight of the feasible structure that meets the stress requirement.

It seems that the depth is a 'size' so that a sizing optimizer can be directly used for its optimization. It cannot. It is well known that among the three areas of structural optimization, sizing optimization optimizes either cross-sectional areas in discrete structures or plate thickness in continuum structures.

The half-model dimensions are the following: end sheet height, 565.15 mm; width, 508 mm; corner post width, 50.8 mm; channel height, 139.7 mm; spacing between channels and top/bottom spans, 95.25 mm. The stress limits are −345 MPa and 345 MPa. The thickness bounds are 2.66 and 12.7 mm. Young's modulus is 199 949 MPa, and Poisson's ratio is 0.3.



(a) (b)

Fig. 4 FEA model used for pilot tests (in inches)

After the problem was defined, an initial model was created on the basis of topology values similar to the real design. The feasible solution was found to meet the stress requirement by manual adjustment, and the sizing-optimal solution was found by the sizing optimization module in MSC/NASTRAN for Windows (FEA program). Then, an experimental design by LHS was done to sample six points from the design space set by the bounds (Table 1). Six points were based on the requirement of quadratic polynomial fitting. The six finite element models were created and optimized for the minimum weight by changing the sizing variables or the thickness of the end sheet plate, the channels and the post, with the stress constraints.

This end structure has some unique features. Usually, when thicker plates are selected, the stress will decrease. That does not necessarily happen to this structure. If thickness is addedto one part, e.g. a channel, the stress in the end sheet could increase, instead of decreasing as initially thought. Manual

Table 1 Topology designs and weights

Sampling points	Order	X_1 (mm)	X_2 (mm)	Weight w (kg)	Savings (%) based on 21.8
Initial model before size optimization	1	25.4	25.4	21.8	
Initial model after size optimization	2	25.4	25.4	18.9	
Group 1, 6 points	1	12.7	38.1	18.8	
	2	0.0	50.8	16.8	
	3	44.5	19.1	17.2	
	4	31.8	12.7	18.3	
	5	19.1	6.4	24.1	
	6	38.1	25.4	16.9	
	Optimal	27.4	50.8	12.8	41
Group 2, 15 points	1	31.8	6.4	19.6	
	2	19.1	50.8	20.5	
	3	25.4	38.1	14.5	
	4	50.8	31.8	22.4	
	5	44.5	25.4	16.3	
	6	12.7	38.1	18.8	
	7	38.1	19.1	17.6	
	8	25.4	6.4	24.7	
	9	38.1	12.7	17.6	
	10	12.7	12.7	25.5	
	11	0.0	44.5	18.5	
	12	38.1	44.5	11.5	
	13	6.4	0.0	24.7	
	14	31.8	19.1	18.3	
	15	6.4	31.8	21.5	
	Optimal	50.8	50.8	10.7	51
Group 3, 6 points	1	25.4	50.8	14.7	
	2	12.7	31.8	21.9	
	3	19.1	0.0	23.3	
	4	6.4	12.7	24.9	
	5	38.1	38.1	13.7	
	6	44.5	25.4	16.9	
	Optimal	50.8	50.8	10.7	51

adjustment may make the situation increasingly worse, i.e. increasingly higher weight. It was not until the sizing optimizer was used that this phenomenon was noticed. The sizing optimizer finds the feasible and optimal result at the same time, much faster than manual adjustment. The six results correspond to the optimal weight of the six topology designs, optimal in terms of sizing or plate thickness.

A kriging model was built to fit the data to obtain the approximate weight response against the topology variables. An optimizer of SQP was employed to find an optimal solution or design. Based on this new topology, another FEA model was created and sizing optimized. Its weight is lower than any design sampled, by 33 per cent compared with the initial model with sizing optimization and by 41 per cent without sizing optimization. The larger portion of the saving is due to topology optimization.

Next, 15 more samples were evaluated with the same procedure: using FEA to find sizing-optimal results, fitting the kriging model to the data, and optimizing by SQP for the optimal topology. The solution went through FEA sizing optimization and the best result in terms of both sizing and topology was found, which is now lower by 45 per cent than the initial model with sizing optimization and by 51 per cent without sizing optimization. It is better than any design already tested for building the response surface. Again, topology optimization plays the more important role.

The response surfaces against the two topology variables are shown in Figs 5, 6, and 7, representing the models from six samples, 15 samples and all 25 samples respectively including all the tests and the optimal solutions. Since inches and pounds force were originally used as the units in the modelling,

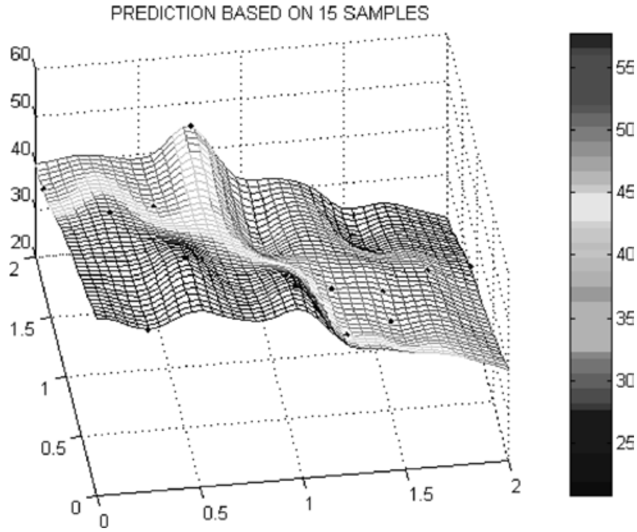


Fig. 6 Response based on 15 samples (depth in inches; weight in pounds-force)

optimization, and output plots, they need to be kept and are shown in the figures. The model based on six samples was cross-validated by itself (the first group); about 10 per cent were left out for building the models. The model based on 15 samples was cross-validated by itself (the second group) and validated by the first group (six samples). The empirical r.m.s error, maximum error, and relative r.m.s. error, i.e. r.m.s. error divided by mean weight, with zero-, first-, and second-order regression polynomials and Gaussian correlation models are listed in Table 2.

Later, a new six-sample group with the lowest C (equation (2)) among 5000 LHS sample groups was found and the FEA models corresponding to the six

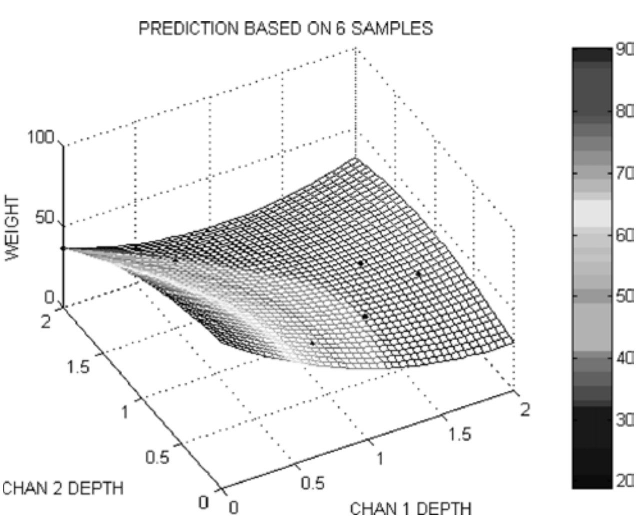


Fig. 5 Response based on six samples (depth in inches; weight in pounds-force)

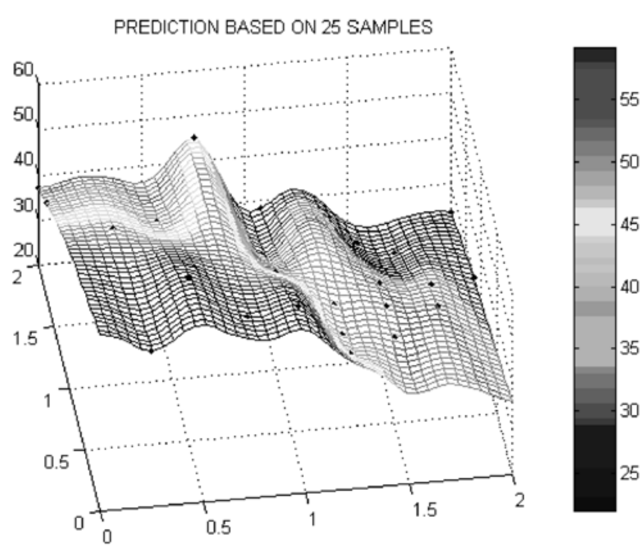


Fig. 7 Response based on 25 samples (depth in inches; weight in pounds-force)

Table 2 Validation result

	R.m.s. error (kgf)	Maximum error (kgf)	Relative r.m.s. error (%)
Group 1			
Cross-validate using group 1			
regpoly1	2.25	4.28	12.0
regpoly0	3.07	6.30	16.4
Group 2			
Cross-validate using group 2			
regpoly2	4.35	9.07	23.0
regpoly1	2.75	6.94	14.6
regpoly0	2.54	4.76	13.5
Validate using group 1 (model based on group 2 only)			
regpoly2	1.45	3.21	7.8
regpoly1	1.35	2.98	7.2
regpoly0	1.71	3.76	9.1
Group 3 (new group)			
Cross-validate using group 3 (new group)			
regpoly1	2.18	3.58	11.2
regpoly0	1.86	2.59	9.7
Validate using group 1			
regpoly1	1.63	3.40	8.75
regpoly0	2.93	6.40	15.7

tests were optimized in terms of sizing. A kriging model was built from the data and then was optimized by SQP. The surface plot is shown in Fig. 8.

The cross-validation by this group and validation using group 1 data mentioned above show the lower relative r.m.s error values (9.7 per cent versus 12 per cent). R.m.s. error, maximum error, and relative r.m.s. error for cross-validation are listed in Table 2. The optimizer found the global optimum design (50.8 mm is the limit) with the minimum weight in the design space as shown before. Using only six points, this sampling produced the best solution found using 15 points by the previous sampling.

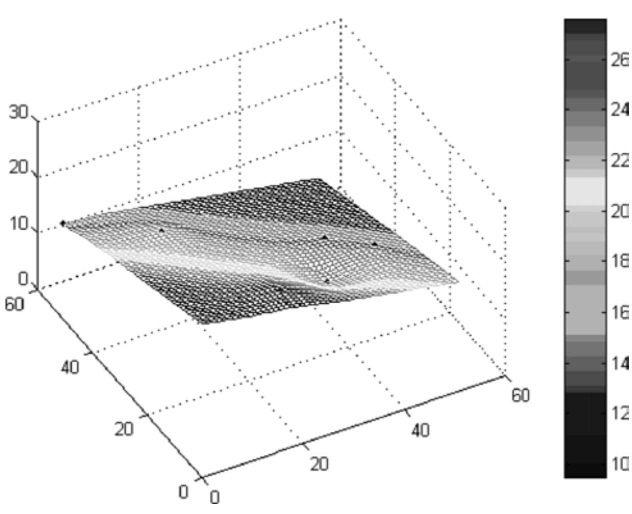


Fig. 8 Response based on six new samples (depth in inches; weight in pounds-force)

The pilot tests show that HIMO is practical and can result in significant benefit. Although it is desirable that the response surface is sizing-optimal so that the topology optimization is more meaningful, sizing optimization might take a long time to run for large FEA models. In those cases, the feasible surface could be used to deal with the major performance constraints. After upper-level optimization is carried out on the feasible surface, the topology-optimized solution can go through sizing optimization to find the final sizing- and topology-optimized design. Of course, optimizing optimal surfaces may result in optimal solutions, and optimizing feasible surfaces may result in suboptimal solutions.

4 REAL APPLICATIONS

HIMO was used to optimize the end structure layout of a boxcar, which is the whole end structure from which the bottom portion was used in the pilot tests mentioned above. The existing structure has an end sheet and seven channels plus a corner post as reinforcement. From previous experience and knowledge, channels are very efficient when compared with many other shapes for the similar

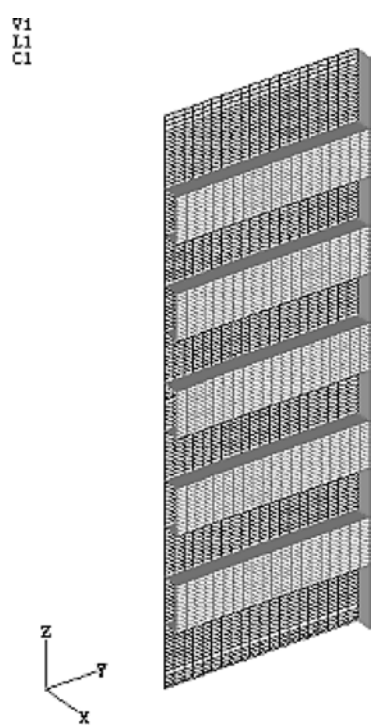


Fig. 9 Car end structure (half)

loading conditions. So it was decided to keep using channels for reinforcement. The corner post also serves other purposes beside reinforcement; so it was kept also.

The layout questions include how many channels should be used, what channel depths and heights should be used, and where the channels should be located. The sizing variables are the thickness of the end sheets, the channels, and the post. The goal is to minimize the structure weight while satisfying the stress and buckling constraints as well as minimum thickness and discrete thickness values corresponding to the sheet metal gauges. The topology constraint is that the sum of the channel heights and spacing between them plus the top and bottom spans is equal to the space allowed. The end structure assembly is loaded with pressure, uniformly distributed on the end sheet with two load cases.

As for the boundary conditions, since HIMO needs many small models to evaluate many options quickly and reliably, the whole vehicle model was compared with a small model that has only the end sheet, the channels, and the corner post. For the latter, as in the pilot test, the borders are pinned without any displacement but can rotate. Only half of the structure needs to be modelled because of the symmetry of the structure, loading, and boundary condition. Therefore, a symmetric constraint was applied to the centre-line to simulate the other half. Later, for buckling analysis, the whole end structure was modelled to guard against possible non-symmetric modes. The initial comparison of the displacement and stress output between the accurate model of the whole structure and the end-only model seemed to show quite close results but, when buckling results were compared, the results from the simplified model did not provide even trend guidelines for the real behaviour from the accurate model. After several trials, it was found that the model of one sixteenth of the whole structure provides very accurate results. The time used to analyse the model was reduced to about 10 min from more than 4 h for the run for the quarter-model (with a 600 MHz computer; it could be much faster with a newer computer).

A previous similar project with lower loading resulted in five channels by HIMO. Although the load values are quite high in this new project, it was suspected that five channels still could meet the entire requirement. Also, from the previous project, the present authors learned that, when the channel depths are allowed to reach the highest limit, the stress can be reduced and buckling eigenvalues can be increased considerably, with only a slight increase in weight. So the depth would be the upper limit. Finally, five variables representing five channel heights and five spacing variables remained, resulting in a total of ten topological variables.

After the problem was formulated, the first task was to explore the whole topology design space to locate the feasible subspace where the designs can satisfy the buckling constraint without changing the designs that can satisfy the stress constraint, to keep the weight low. Using the new sampling method (Latin hypercube design with maximum sum of distance and minimum CL_2 discrepancy (LHMDmd)), 20 samples were designed that meet the topology constraint requirement. The intention was to try only ten samples to explore the whole space quickly. If they turned out to be feasible and the response surface needed to be built, the other ten samples could be tested, resulting in 20 sampling points for the surface.

Ten tests were carried out. Without changing the plate thickness of the end sheet and the channels from the sizing optimization outputs, the models were run to determine buckling eigenvalues. Some were far below the limit, whereas the others were close. The sampling plan or topology designs were sorted according to the eigenvalues. The trend became clear that, when the channel height is increased (within some limits) and the spacing between the channels is reduced, the eigenvalues become higher. The possible feasible subspace was set by specifying the bounds for the topology variables. Another group of 20 samples was designed using LHMDmd. Several trial testing results show the subspace was still not small enough since the eigenvalues were still not close enough to the limits. Further reduction in the subspace was made. New samples were designed and tested. After several more trial samplings, the near-feasible subspace was found. The plate thickness either does not need to increase or needs to increase only slightly in order to meet the buckling resistance requirement. Therefore 20 samples were planned and tested. All resulted in feasible designs in terms of the stress and eigenvalue constraints and optimal designs for sizing variables.

The kriging surface with the zero-order or constant-regression polynomial model and the Gaussian correlation model was fitted to the data. The cross-validation shows that the r.m.s error is 10.2 kgf, the maximum error is 20.9 kgf, and the relative r.m.s. error (i.e. r.m.s. error divided by the mean of the weight response) is 2.47 per cent. Then, a genetic algorithm followed by SQP was used to find optimal topology.

Since the relative r.m.s error was still not very small, a promising subspace was sought to confine the region further to be explored. The 20 samples tested were sorted by weights of the assembly, and the subspace defined by the designs with the ten minimum weights was used as the promising subspace for the next sampling group.

Ten more samples were designed and tested. Together with the ten best samples in the last group, 20 samples were used to build the kriging surface with the constant-regression model and Gaussian correlation model. The cross-validation results are the following: the r.m.s error is 4.7 kgf, the maximum error is 12.9 kgf, and the relative r.m.s error is 1.16 per cent. The genetic algorithm and SQP found the optimal solution that is quite close to the best solution tested in the newest group, which is also the best of all the options tested. Since this new optimal result is not significantly better than the last, only about 13.6 kgf lower, and, since much better solutions were not expected from experience, sampling and modelling did not continue.

For the end project just described, the final design was made on the basis of the best solution that went through final sizing optimization after topological optimization, resulted in a weight saving of over 907 kgf, or about 36 per cent (18 per cent was obtained for the previous project). Both real designs also meet several other functional and manufacturing requirements and overload concerns. Again, using five channels instead of seven considerably reduces manufacturing cost for the company. For proprietary reasons, the details of the final designs are not provided here.

5 CONCLUDING REMARKS

The key features of HIMO are as follows.

1. All the performance constraints are dealt with at the sizing optimization level only, greatly reducing the effort to optimize topology.
2. The number of topology design variables is much smaller than used in many other topology optimization approaches.
3. The sizing optimizer is used to find the feasible and optimal design among all the sizing options nested within one topology option, for every sampled point.
4. The response is an optimal surface in terms of sizing or plate thickness in continuum cases, and topology optimization then works on this surface to find the best topology design.

Both the pilot tests and the real applications show that the HIMO approach works for real layout optimization. HIMO offers another heuristic and systematic approach for addressing difficult layout optimization problems for real structures and is very flexible. When the budget is tight, less sampling and testing can be done and still a significantly improved design can be found. When the budget allows more work, and finding the optimal result is very important, more sampling and testing can be conducted, resulting in much better or optimal solutions.

This paper has discussed the present authors' initial investigation of HIMO that shows its applicability to practical problems. Research is currently under way to improve its effectiveness and efficiency further. This research will also include testing the design optimization of a large-scale system. Many details regarding metamodelling and optimization strategies for HIMO will be reported in subsequent papers.

Another new approach proposed by the present authors [**24**], sizing optimizer for topology optimization (SOTO), directly uses a sizing optimizer to optimize topology. The successful application of SOTO for optimizing railcars shows that it can be effective and efficient too for real design optimization.

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APPENDIX

Notation

CL_2	centred analytical L_2 discrepancy
d_{ij}	Euclidean distance between points i and j
$f(x)$	known polynomial function of x, a global regression model, often a constant term
n	number of samples
P_n	set of n points
s	number of dimensions
U_{ij}	jth element of an independent $U[0, 1]$ (uniform distribution between 0 and 1) random variables, independent of the π_{ij}
X_{ij}	jth component of the ith sampled point
$y(x)$	unknown function of interest
$Z(x)$	correlation model
π_{ij}	jth element of the ith independent uniform random permutations of the integers 1 to n