Excerpted from

Societal Systems,
Planning, Policy, & Complexity

J. Warfield, Wiley, 1976

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Interpretive Structural Modeling

A process called interpretive structural modeling (ISM) was introduced in [1], and relevant objectives were discussed in Chapter 7. In ensuing chapters, underlying mathematics and specific methodology based on the mathematics were introduced. In this chapter, an aggregated description of the ISM process is given. The process has many *detailed ingredients which* themselves constitute a rather extensive structure. In the evolution of the structure of the process, four prominent elementary types of components are the events that occur in the process, the activities that are carried out, the specific decisions that are made, and the logical connections among these process ingredients. Responsibilities for the various activities and decisions are assigned. The actors to whom these assignments apply are the planner of the ISM exercise, the leader of the exercise, the developer of the structural model (which often is a group), and the computer.

Decisions assigned to the computer include those that require mathematical operations which cannot readily be done by the leader or the developer, and those that relate to the intrinsic character of the ISM process. The latter primarily involve the sequencing of activities and decisions.

Decisions assigned to the leader are limited to those that require an understanding of the ISM process, coupled with an understanding of group process as it relates to structuring.

Decisions assigned to the developer of the model include all those that involve the substance of the theme of the exercise, as well as certain decisions concerning the way in which the results of the effort will be displayed.

The assignment of decisions is made in such a way that the computer, acting in a facilitating role, is effectively in charge of the sequence of the exploration. The developer retains final control over the output of the process. The leader is primarily in an expediting role and in most instances will not become involved in discussions of substantive issues related to the theme of the exercise. The leader must be able to make certain judgments concerning when to terminate a session and must make judgments concerning alternative methods to be applied at several points in the exercise.

A considerable amount of preliminary effort is required to prepare for that portion of the process which involves computer assistance and related facilities. This effort is critical to the successful conduct of that portion of the process. It may be carried out by a planner who would not necessarily be involved in the exercise, but can also be carried out by the person who will lead the exercise. The planner should have a very thorough knowledge of the ISM process, obtained both by study of the process and by participation in several sessions.

Major Event Sequence

The sequence of major events in ISM is shown in Figure 14.1. These events are assigned numbers for convenient reference.

The event sequence begins with E-1, the selection of the theme of the exercise. A theme, which can generally be set forth in a few sentences of prose, provides general orientation to the substantive area of exploration. It will relate to an object system to be explored, but the object system is usually not well understood, which is why the structuring exercise is to be conducted. Hence it is not possible to begin with a definition of the object system itself—that is an aim of the exploration.

Next the developer of the structural model is identified. The developer will normally be a group of people who are informed about various aspects of the theme to be structured, and motivated to explore the theme. When the developer is identified, the elements and contextual relation or relations to be structured can be selected. The selection of elements may very well involve the developer. Two common methods of element selection are the brainwriting pool (with editing) described in Chapter 2, and content analysis of selected documents. The latter can be specially prepared prose discussions of the theme furnished by the developer and may include research papers or other sources of information. The choice of the contextual relation or relations should be made by the planner of the exercise with advice from the developer. There should be specific consideration of a contextual relation to determine that it is transitive, or that it is "sufficiently" transitive that the exercise is likely to be useful.

There is no guarantee that the ISM process will be applicable to a contextual relation chosen at random or chosen because it is relevant. The process is based on the assumption that the relation is transitive. Experienced scientists have sometimes failed to consider carefully whether a selected relation is transitive. If the model developer is not acquainted

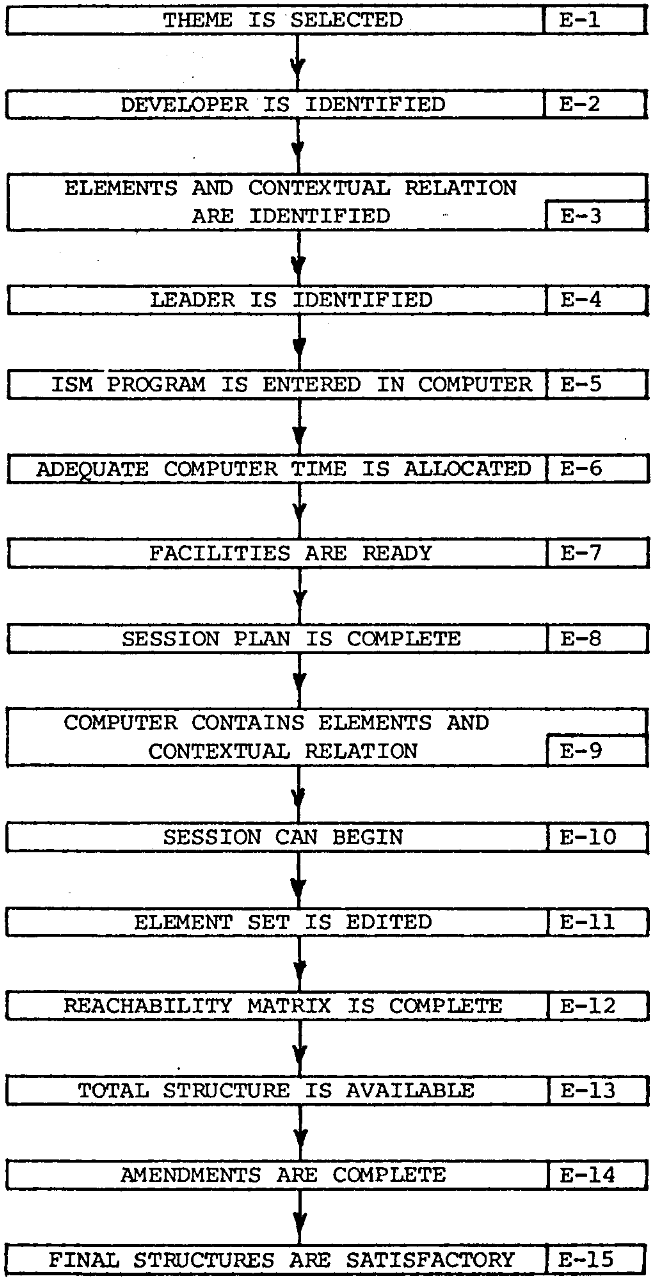

Figure 14.1. Major event sequence in ISM.

with the meaning of transitivity, assistance will be needed in determining whether the relation is transitive. It has frequently been found possible to construct a transitive contextual relation that is quite satisfactory in exploring a particular issue in instances where the original choice was not very satisfactory.

Even when it cannot be demonstrated that a contextual relation is inherently transitive, it may be quite feasible to employ the assumption of transitivity to develop an initial structure. But if such a course is followed, it should be done knowingly.

The choice of leader should be made with a view to the importance of group facilitation, but the leader also should have a good knowledge of the ISM process. The leader should understand that the exercise is being carried out to learn through development of a structural model, not to provide the leader with an opportunity to display his knowledge of the ISM process. Exercises can easily be deflected from the main purposes by overdiscussion of details of the process. It is important to remember that the process is not intended to be a training course in methodology. It has been carefully designed to accommodate individuals who have no knowledge of the relevant mathematics, may be extremely busy and do not want to waste time in irrelevant discussion, and do not want to be embarrassed by being asked to make decisions about matrix algebra, set theory, or other specialized topics of which they are not knowledgeable.

If there are individuals in the group who desire additional knowledge of the ISM process beyond that needed to participate productively in an exercise, it should be made available at an appropriate time when the process is not being carried out.

It is necessary to put the ISM program in the computer and to make sure that adequate computer time has been allocated for its use. Since the process may be carried out in a computer time-sharing mode, priority on access to the computer should be investigated in planning the exercise. It is inappropriate to have a group idly waiting for access to the computer, and it is disastrous to be disconnected before the exercise can be completed.

A photograph taken during the conduct of an ISM session is shown in Figure 14.2. The presence of appropriate television display screens is readily visible. No computer is seen, since the computer will normally be accessed remotely through a telephone line connected to a computer terminal. The computer terminal is not visible in Figure 14.2, and it is generally desirable that a low profile be maintained for the terminal. It is there as a means of facilitating the group process and not to serve as an impudent reminder of the limitations to unaided human intellectual effort.

When the facilities are ready, it is appropriate to review the planning for the ISM exercise with emphasis on the first session. Elements of planning are discussed on pp 350–354. When the planning is completed, the elements and a contextual relation are entered in the computer and the session can begin.

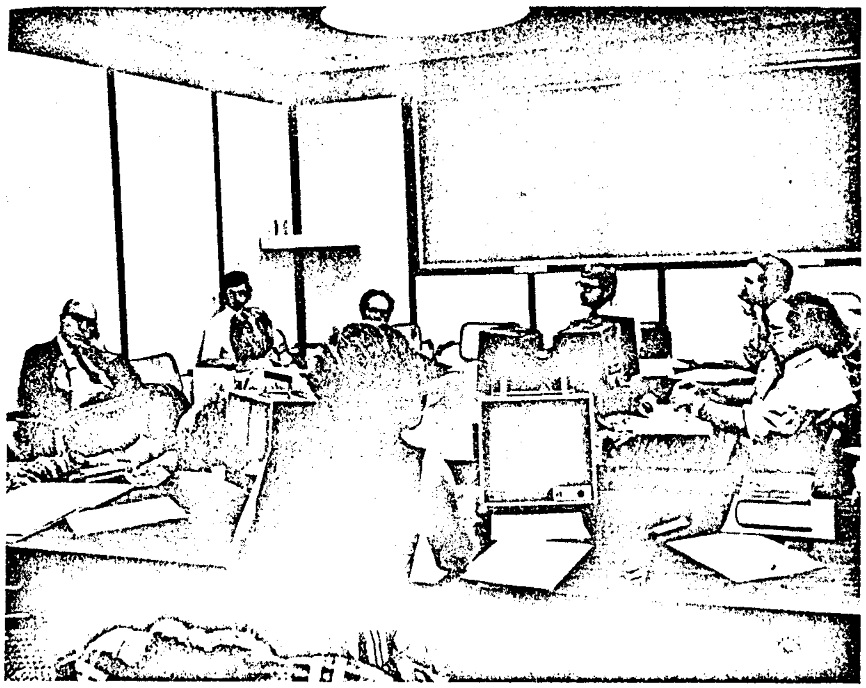

Figure 14.2 ISM session in progress. Photograph courtesy of Battelle Memorial Institute.

If the element set requires editing, as will be true in many instances, this editing is carried out with machine assistance. Once the editing is complete, the transitive embedding process described in Chapter 12 is initiated. The result of transitive embedding is the development of a reachability matrix model.

The computer can develop the total structure of the reachability matrix using the methods described in Chapter 10. The developer then examines the structure and makes such amendments as are desired. The amendment process may include several important decisions by the leader, some of which require intimate knowledge of the ISM methodology. As a result of the amendment process, it may occur that more than one structure is produced, as explained in a subsequent section on amending structural models. When the final structures are satisfactory the process is complete, although documentation and prose interpretation should frequently follow the completion of the process.

Editing the Element Set

Experience has shown that while a given set of elements may be regarded very highly by participants before an exercise begins, the process of responding to machine queries involving paired comparison of elements with respect to the selected contextual relation is very effective in revealing discrepancies in the wording of the elements. Also, in the process of responding to queries, it often occurs that additional elements are conceived which were not recognized previously as relevant to the exploration.

It is quite possible that a set of elements may already have received editing during a previous ISM exercise and additional editing may not be needed. But if the set of elements is being explored via ISM for the first time, the chances are very high that editing will be needed. It is even possible that the contextual relation will also be changed during the editing, though this has not been common in a variety of exercises conducted in the past.

Therefore, it will generally be appropriate to plan that the editing of the element set will occupy some of the time available to conduct an ISM exercise.

It is clearly undesirable to invest large amounts of time in developing a matrix whenever the element set requires revision. It is desirable to begin the exercise with a procedure that allows all the elements to be examined in comparison with at least one other element. Additional effort may be built in when substantial editing is required. If the number of elements in the set is large, say 40 or more, it will probably be desirable to plan to have at least one session simply for editing.

The scanning method of transitive embedding includes, as the beginning of the first phase, a part wherein every element is examined in relation to one other element. It is possible simply to employ the first portion of the scanning method as a means of editing the elements. If a record is kept of responses to queries, part of the results of the editing exercise may be retained and used to initiate the matrix filling. If it should turn out that no editing is required, all of the results of the computer questioning can be retained as useful matrix data.

When the first phase of the scanning method is begun, a partition is constructed based on one element in the set, as explained in Chapter 12. All other elements are compared with this single element. It has been argued that the results of the exercise may be biased by the heavy emphasis given to the single element. Further it has been said that it is somewhat boring to see the same element repeatedly.

Subjectively it appears that if the single element against which all others are compared is chosen from the set of elements using the criterion that the element seems likely to appear near the middle of the anticipated structure, the editing process may be both more efficient and more interesting than if the element is near the extremes of the structure. This criterion is useful only if the structure has several levels. Since the structure is unknown and remains to be developed, the iterative nature of structural exploration clashes with the desire to employ the structural knowledge to be attained before it becomes available.

No empirical evidence supports the contention that the ultimate product of an ISM exercise is significantly distorted by the choice of the element on which the first partition is made in the first phase of the scanning process.

When editing is underway, the leader may be called on to help stimulate discussion if the participants are engaging in their first ISM session. The learning aspects of ISM are most beneficial when the participants discuss each response to computer-initiated inquiries before a vote is taken. While it will be possible to amend the initial structure and, as part of the amendment process, to change one or more of the elements, it is clearly beneficial to have a well-edited set, since substantive amendments may not be necessary with a well-edited set and the ISM exercise will be shorter.

Leadership of ISM Exercises

The role of the leader, which is critical in ISM exercises, can be explored in terms of both its positive aspects and its potentially negative aspects.

In the positive sense, the leader is a facilitator, and all leader actions should be assessed in terms of whether they will facilitate the work of the group. A good working knowledge of the ISM process details is needed to provide this facilitation, but it is also desirable to know quite a bit about group processes.

If the leader is expert in the substantive material related to the theme of the exercise, great skill will be required to suppress such knowledge in exercising leadership. A person who does not desire to do this should probably be in the role of developer rather than in the role of session leader. In general, the leader has to be neutral with respect to the substantive task effort. All leadership energy is needed for facilitating the group effort.

The leader and planner should work together to help ensure that the technology or process facilitates rather than detracts from the substantive task. The first form of debilitation in this respect can occur when the start of the session is delayed through ineffective preparation of the technology. The first impression is unfavorable, and the process gets off to a bad start or does not start at all. The leader is perceived as one who does not value the personal presence of the participants enough to have the process ready when they are ready.

A second form of debilitation occurs when the participants become involved in an adversary relationship with the ISM process itself. This can easily happen if a leader suggests that the process is discovering their "mistakes," or seeks to place on them cognitive tasks that they are not equipped to handle in order that they accommodate to the process. No option should be suggested to a group unless adequate provision has been made by the leader for a means of group attainment of that option. Such suggestions are serious errors in strategy, as they suggest to the participants that they are somehow inferior to the leader or to the machinery. This introduces justifiable emotional responses that detract from the quality of the substantive task effort. Such matters are inconsistent with the character of the ISM process, which has been designed to serve the task and not to interfere with it.

If the leader suggests to the group that they can change the contextual relation, it is incumbent to provide a means for doing so. For example, a small group might convene apart from the session to consider this possibility.

The option to make amendments to a total structure should not be suggested unless there is adequate technology or other effective means for carrying out such amendments as a group effort. Individuals who wish to make arbitrary changes in the structure produced by a group can certainly do so following the process if they wish. But there is a certain integrity to the group effort that must be sustained as a part of the validity of the ISM process. The leader is responsible for sustaining this integrity.

In responding to questions about the ISM process, the leader has an opportunity to display extensive knowledge about its workings. But usually such responses simply lead the group into unfamiliar areas. Among the useful responses that do not necessarily lead into unfamiliar areas can be anecdotes about prior exercises that illustrate features of the process, emphasis on its open character with power remaining with the group to convene later to change the output if they wish and suggestions as to how this may be accomplished, emphasis on limitations placed on the process by the scope and purpose of a particular exercise, and what can be expected in the product in particular constrained situations. Also opportunity can be extended to explore the process details in whatever depth the participants desire in a setting convened for that particular purpose. Examples of products of previous exercises may be discussed to help illustrate the nature of the anticipated product. Mead's expectation principle (Chapter 3) is relevant.

A major function of planning and leadership is to attempt to anticipate how many sessions are required and to estimate their duration. Another important function is to decide on the choice of transitive embedding method (see Chapter 12) and to decide whether a presentation should be in levels or in stages (see Chapter 10). These considerations are discussed in the next two sections.

Estimating the Number of Sessions

It is not possible to predict accurately how much time will be needed to edit an element set, to fill a reachability matrix, or to amend a structural model. While the process does save time in carrying out these operations, the amount of time needed is dependent on the amount of discussion among the participants, which bears a close relation to the amount of learning that takes place during an exercise.

A plan for conducting an exercise will normally include a partition of the exercise into sessions. Experience suggests that a session should not occupy more than three hours, though some participants have enthusiastically taken more than this amount of time.

The time for editing an element set will generally consume a single session, assuming the set contains between 20 and 40 elements. If the number of elements is substantially greater than 40, it is prudent to anticipate that more than one session may be needed for editing.

The time required to fill a matrix enroute to the initial multilevel structure depends on at least six factors. Subjectively, the time required will be kept to a minimum by achieving a high quality element set, a high quality contextual relation, effective preparation, and good leadership. Assuming these four factors to be very good, a rough rule of thumb stemming from experience suggests that the time in hours to conduct the transitive embedding and arrive at an initial multilevel structure (before any amendments are made) is approximated by

$$T\,(\text{hours}) = \frac{1}{600}\,e^2p^{0.5} \tag{1}$$

where e is the number of elements in the element set and p is the number of participants engaged in the model development process. Applying (1) to an exercise involving 25 elements with 4 participants, it would be expected that a little over 2 hours would be consumed by the transitive embedding and initial multilevel structure development. Estimates from (1) are subject to considerable error.

The amount of time needed to amend a structural model will vary a great deal, and no formula can be given to assist in estimating the time. The desirability of amending a model will vary greatly with circumstances, and is in the hands of the developer.

Choice of the Transitive Embedding Method

Information is embedded in structural models by transitive embedding and by weighted embedding. Initially transitive embedding is used, as discussed in Chapter 12. Two methods, the scanning method and the coupling method, were described. The scanning method has the advantage that it allows for early appearance of every member of the element set in sequence in questioning, which is particularly useful in editing. Also in the first phase of the scanning method, the matrix to be filled is divided into submatrices, which may provide a useful way of organizing the subsequent activity.

The coupling method is not as advantageous in these respects, but when iterative bordering is used it does permit the process to be stopped at any of a large number of intermediate points for examination of completed substructures. The scanning method also permits examination of substructures at selected points, but there are many less such points than with the coupling method.

It is recommended that the scanning method be used for editing the element set. If an element set is thoroughly edited, the coupling method may be more appropriate, but additional experience with large matrices is required to permit a thorough comparison of the relative merits of these two methods. Whichever method is used, the ISM process can always be interrupted at any point and resumed from that point at a later time, assuming the program status is preserved at the point of interruption.

The choice of method is most significant when an initial editing exercise has been completed with the scanning method and there is a question of whether to start over or continue. If editing changes have been minor, it may be decided to continue the process until a multilevel structure becomes available, using the scanning method. Alternatively, it may be decided to start over, whereupon it may be more appropriate to use the coupling method. The decision will be based on the amount of time available, the schedules, the quality of the information already available in the machine, the size and structure of the matrices, and details of computer programs, such as whether it is permitted to make simple substitutions within element statements without disrupting the process excessively.

Computation of a Total Structure

Whichever method is chosen for transitive embedding, the computer will normally supply a significant fraction of the entries, inferred from the transitivity condition. A typical percentage for entries supplied by the computer is 70 percent, but in one instance involving a 45 x 45 matrix, the computer supplied 83 percent of the entries.

When the transitive embedding process is complete, the computer can extract from the matrix the information needed to define a structural model, arranged either according to the level partition or the stage partition, as discussed in Chapter 10.

The decision as to whether the structure will be arranged using the level partition or the stage partition will normally be made in advance of the exercise, based on the nature of the contextual relation, as discussed in Chapter 10.

Once the reachability matrix is partitioned, it is possible for the computer to identify any maximal cycle sets that may be present. The machine can then represent each maximal cycle set by a single proxy element and construct a condensation matrix, that is, a matrix representing a hierarchy in which each maximal cycle is represented by a single proxy element.

If it occurs that there are no maximal cycles, there is no change required in the reachability matrix. Otherwise, the original reachability matrix permits derivation of a smaller reachability matrix from which the skeleton hierarchy can be constructed.

In preparation for construction of a minimal edge digraph map that preserves reachability, the computer finds the skeleton matrix, that is, the proper $(\ell-1)$th root of the condensation matrix, using the theory given in Chapter 10. Once this matrix is computed, the computer has sufficient information to define the skeleton hierarchy.

Depending on programming details, either the developer or the computer may, at this point, convert the skeleton digraph to a hierarchical interpretive structural model. The computer may supply a list of elements at each level, a list of edges for the digraph map, and a list of the maximal cycle sets, or it may go so far as to construct the complete structure, depending on the availability of a suitable graphical facility.

Examination of a Total Structure

When available, the initial structure can be examined by the developer to see whether it is satisfactory, or whether it requires various types of amendment. This decision will normally be based on three considerations: the interpretability of the structure as it stands, the assessment of the accuracy of the structure, and the desirability of supplementary structures. Interpretability will depend on the number and size of cycles that may be present in the structure. Accuracy will depend on whether the information supplied in developing the model was accurate and the amount of learning that took place during the embedding. The desirability of supplementary structures will depend on the anticipated use of results and the size and organization of the structural model.

Examination of the total structure will be greatly facilitated if the structure can be converted by the machine from the basic model to the interpretive structural model, that is, whether the machine can draw the interpretive structural model, including all the structural features. In the absence of such a capability, the total structure will be arranged manually for examination. This can be facilitated in several ways. For example, if a magnetic wall is available and the elements are attached to portable magnetic cards, a structure can rapidly be constructed from the computer printout, and edges can be drawn with marker pens or chalk.

Amending Structural Models

Structural models may be amended to correct substantive defects or to facilitate interpretation. Amendments of the former type are called *substantive amendments*, while those of the latter type are called *format amendments*. In making either type, it is highly desirable to work either with a single maximal cycle or with a hierarchy. It is important to follow certain sequence rules in making amendments. For example, it is undesirable to make format changes in a cycle or hierarchy if it is known that substantive changes are needed, since the latter will often change the structure. The presumed benefits of format changes may be nullified by the changes in structure induced by substantive changes.

Amendments to Cycles

When the maximal cycles in a total structure have been identified, it is appropriate to examine each maximal cycle with a view to its accuracy and its graphical portrayal. Since the reachability matrix for a cycle is universal, the complete portrayal of a cycle based on the reachability matrix will be a digraph map having an edge directed from each element to each other element. This type of portrayal often is hard to interpret.

For this reason, a procedure is desired that reveals information concerning the adjacency structure of a cycle. This type of information can be embedded in a matrix using the weighted embedding method described in Chapter 13. As mentioned therein, a series of binary matrices can be machine computed from the weighting matrix, each of these corresponding to an adjacency structure called a *threshold structure*. Each weight corresponds both to a threshold and a threshold structure. One particular weight corresponds to a cycle threshold except, possibly, in the unlikely event that changed perceptions lead to weights that do not even produce a cycle when the minimum weight is employed as a threshold. Should the latter occur, an amendment to the total structure is required. It can be achieved through cycle clipping, discussed on p. 360.

Otherwise, the machine can develop the adjacency matrix for the cycle threshold map, and print this information for use in developing an adjacency map for the maximal cycle. This model can be inspected to see whether it is readily interpretable. If it is, the cycle is considered to be *resolved*, and the structure so produced becomes available as an integral part of the total structure. If it is not, two cases remain open for consideration. Either there is a threshold cycle that is not interpretable as it stands, or there is no threshold cycle. In either case, the question as to how to represent the adjacency within the cycle can first be addressed by examining the structures that evolve from the set of threshold adjacency matrices. This possibility can be explored by asking the machine to provide structural information needed to portray all of the threshold maps corresponding to the threshold matrices in sequence, beginning with the simplest structure. It corresponds to the largest threshold.

If one of these structures is acceptable, the original maximal cycle will have been *clipped*, that is, no cycle set in the threshold structure contains as many elements as the original maximal cycle set. In this event, it is recommended that the entire maximal cycle set be disconnected from the total structure by simply removing its proxy element. Then the threshold structure can be interconnected with the rest of the structure using the coupling method of transitive embedding.

When there is no satisfactory threshold matrix, it will normally be possible to choose one that is approximately correct. Then either elements or edges may be added to or removed from this structure by treating it just as though it were a total structure to be amended. If this procedure is satisfactory, the structure obtained can then be used as an adjacency structure for the elements involved, whereupon it can be interconnected with the rest of the structure.

In the event that it is desired to retain the maximal cycle set in its cyclic form, or it is too difficult to interpret using any of the foregoing procedures, it is recommended that an exploration of its geodetic cycles be carried out. The computer can determine the geodetic cycles from the weighting matrix and can construct a hierarchy of these cycles in the manner indicated in Chapter 13. This hierarchy introduces a natural sequence of study of the maximal cycle set, which should assist significantly in its interpretation. Following the exploration of the geodetic cycle sets, the procedures described earlier should be facilitated.

It is possible that none of the foregoing approaches will lend adequate insight into a very complex cycle.

The recommended procedures for cycle resolution described in the foregoing are portrayed in Figure 14.3. To illuminate further these procedures, various amendments are described in greater detail.

Format Amendments to Cycles. An amendment to a cycle that does not affect the reachability matrix of the cycle set is called a format amendment. After completion of a format amendment, the reachability matrix of the cycle set will still be universal.

CONTRACTION. Contraction of a cycle is carried out in the manner described in Chapter 10. The result is to replace two adjacent vertices in the cycle with a single vertex. Or if carried out repeatedly, several vertices may be replaced with a single vertex. The vertex that remains may be renamed and is considered as a proxy for the elements it replaces.

EDGE ADDITION. It may occasionally occur that it is desired to add an edge to a digraph map of a cycle. This has no effect on the reachability, but may be carried out for interpretational reasons. The affected matrix is the adjacency matrix. Addition of an edge corresponds to replacing a 0 in that matrix with a 1.

EDGE REMOVAL. An edge removal from a cycle is a format amendment only if universal reachability is preserved. Removal of an edge corresponds to replacing a 1 in the adjacency matrix with a 0.

REPLACING A MAXIMAL CYCLE WITH A THRESHOLD CYCLE. In Chapter 13, a threshold cycle was defined through a sequence of reachability matrices derived from various operations on the weighting matrix W for the maximal cycle. Specifically, there exists some weight k called the cycle threshold such that if all weights greater than or equal to that weight are replaced by 1 and all other weights are replaced by 0, the binary adjacency matrix that results has a universal reachability matrix. The threshold cycle corresponds to the digraph map of that adjacency matrix. It will normally have fewer edges than the digraph of the reachability matrix.

Substantive Amendments to Cycles. Three kinds of change are considered to be substantive amendments to cycles. These are element addition, element removal, and cycle clipping.

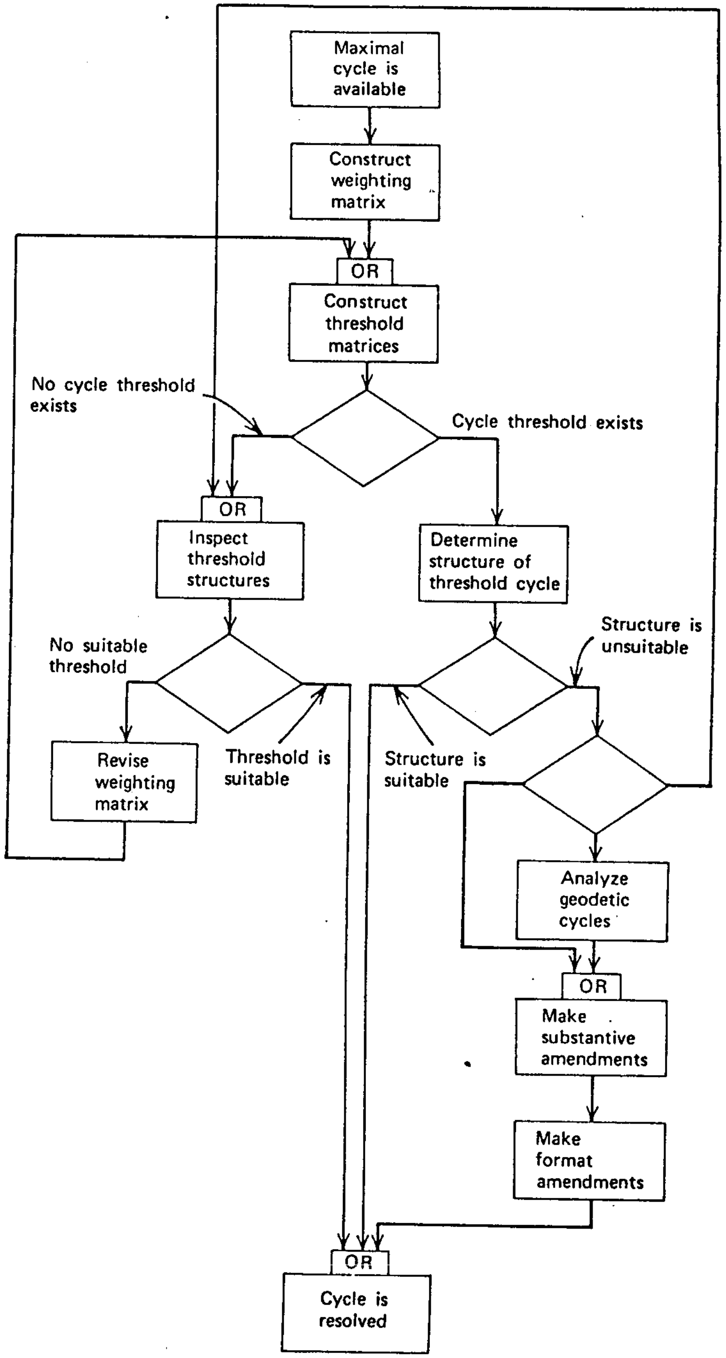

Figure 14.3. Cycle resolution procedure.

ELEMENT ADDITION. If in studying a cycle, it is discovered that an element should be added, the developer should consider two possibilities. If the element to be added is to become part of the cycle, one procedure applies. If the element is to be connected to the cycle in only one direction, then it does not become part of the cycle. In this latter instance, the added element should be dealt with as an amendment to the total structure of which the cycle is a part. Transitive bordering is appropriate as a means of adding such an element.

Hence only the situation where the new element is to become part of the cycle need be considered further. Such an element can be added by simply adding a new row and column indexed by that element to the total reachability matrix. Its row and column will be identical to those of the other elements in the cycle.

ELEMENT REMOVAL. Removal of an element from a cycle is carried out by eliminating the element and any edges that connect to it from the cycle. When an element is removed from a cycle, it is appropriate to remove its row and column from both the total reachability matrix and the weighting matrix, and renew the exploration with the structure found from the new weighting matrix.

CYCLE CLIPPING. A cycle is said to have been clipped if an edge is removed from the cycle and, as a result, the structure that remains is no longer a cycle. Cycle clipping may be introduced deliberately to simplify interpretation. As explained earlier, removal of an edge from a cycle is considered to be a format change, as long as the replacement structure still has a universal reachability matrix. If the latter condition is violated by an edge removal, the cycle is clipped and the change is substantive.

Since removal of an edge from a cycle is equivalent to insertion of a weight of 0 in the weighting matrix of the cycle, the effect of such a removal can be determined by inspection of the threshold matrices obtained from the revised weighting matrix. Then it can be seen whether the change is a format change or a substantive change.

Cycle clipping can be done in either of two ways. The most direct route involving machine assistance is to increase the threshold above the cycle threshold and use one of the threshold matrices that is so found. However, it is also possible to clip the cycle by simply removing an edge without recourse to the threshold structures.

Cycle clipping should not be done without strong justification. Deliberate substantive change by clipping to simplify a cycle may be a cause for very misleading interpretations. If cycles are clipped, the reason should almost always be to improve accuracy rather than to make it possible to interpret the cycle as it stands. If done for the latter reason, cycle clipping should be

viewed as a temporary expediency, which later may be repealed by adding edges after greater understanding is achieved.

If cycles are clipped by using one of the threshold matrices, the selection of a particular threshold matrix completes the process of cycle resolution, except as just noted. If none of these structures is satisfactory, it is recommended that cycle clipping be done by adjusting weights downward in the weighting matrix and repeating the study with a new set of threshold matrices until a suitable structure is found. When this occurs, the cycle is resolved.

Outcomes of Cycle Resolution

Cycle resolution terminates in one of three outcomes. Either the cycle threshold structure found from the weighting matrix is satisfactory, or substantive amendments are made to this structure by adding or deleting elements or edges while preserving the unilevel character of the cycle, or the cycle is clipped and is no longer a cycle (though it may contain a cycle).

If cycle clipping is carried out, a multilevel structure normally results from the cycle resolution process. (It is theoretically possible that the result would be a unilevel structure containing one or more disjoint elements and possibly a cycle smaller than the original one.) The changes produced by cycle resolution then need to be introduced into the total structure of which the original maximal cycle set was a part.

Those elements that remain in a cycle following cycle resolution can be represented by a suitable proxy in the total hierarchy as before, as no change is required in the corresponding row and column. Elements that originally were part of the maximal cycle, but are no longer contained therein as a result of cycle resolution, will be considered as though they were new elements to be attached to the hierarchy, and the first connection entries will be those known as a result of the cycle resolution. Other entries can be made in the manner used in transitive bordering.

Since the cyclic part of the structure that results from cycle resolution is masked when it is reabsorbed in the hierarchy, it is necessary to preserve that cyclic part separately for later use. This can be done by saving the adjacency matrix.

Amending Multilevel Structures

With an understanding of cycle resolution, it is possible to portray a general approach to the amendment of multilevel structures, whereupon the amendment of hierarchies can be treated in more detail.

The reason amendments are usually made either to cycles or to hierarchies is that it is procedurally easier to amend a cycle or a hierarchy than it is to work with a complete multilevel structure that contains cycles. While a multilevel structure is usually available at the beginning, this structure is amended when a maximal cycle is replaced with a proxy. This type of format amendment is the only one that is necessarily made to multilevel structures in their most general form.

When a series of substantive amendments is made to cycles or to hierarchies, these will eventually lead to a replacement of the original multilevel structure.

Figure 14.4 shows the general plan for amending structural models. Beginning with a structure that may be multilevel and contain cycles, initial inspection reveals whether the structure is cycle free. If it is not, each maximal cycle is resolved separately, using the cycle resolution procedures described previously. Upon resolution of any particular cycle, the result will either be that the replacement structure will still be a cycle, in which case its original proxy (or another chosen as a better one) remains in the total structure, or it will be necessary to amend the total structure to take account of the substantive change. In either event, the total structure can be recomputed and again inspected.

If the total structure is a hierarchy, substantive or format amendments may be made in the hierarchy. Following each amendment, it is desirable to have the capability to examine the new structure.

Eventually, it is assumed, a satisfactory total structure will be attained, whereupon the amendment process terminates and the interpretive structural model is available for documentation and descriptive purposes.

Amendments to Hierarchies

As indicated previously, amendments to hierarchies are classed as substantive amendments and format amendments. It is recommended that substantive amendments be made before making format amendments, as a general rule. Format amendments are not intended to introduce new information but simply to make the hierarchy easier to interpret.

Substantive Amendments to Hierarchies. Substantive amendments to hierarchies are of four basic types. An element may be added or removed, and an edge may be added or removed. Removal of either an element or an edge replaces one hierarchy with another. Addition of an element or edge may introduce a cycle to the structure, whereupon it is no longer a hierarchy.

ELEMENT REMOVAL. Removal of an element from a hierarchy corresponds to deleting that element and all edges that are incident with it. This corresponds to deleting the row and column indexed by that element from the reachability matrix for the hierarchy.

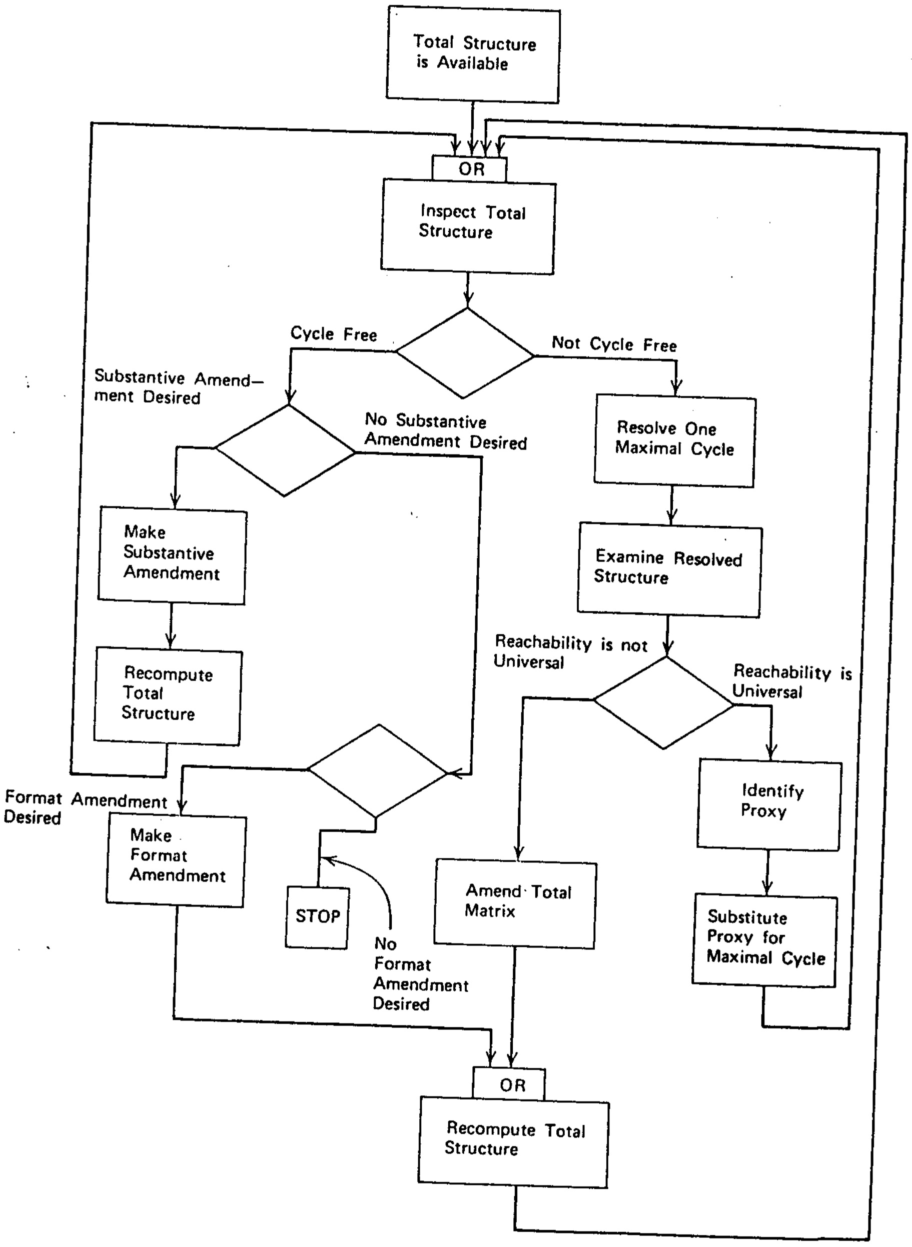

Figure 14.4. Plan for amending interpretive structural model.

It follows from the interconnection theory (Chapter 9) that if a row and column indexed by the same element are deleted from an equally indexed reachability matrix, the remaining matrix is a reachability matrix. Hence a new hierarchy can be computed from the new reachability matrix.

ELEMENT ADDITION. Addition of an element corresponds to inserting a new element in the element set and to interconnecting the new element with the rest of the structural model. The transitive bordering method of Chapter 12 can be used to carry out element addition.

EDGE REMOVAL. Edge removal consists simply of deleting one edge in a hierarchy. To incorporate such a change in the computer, it is not appropriate to work with the reachability matrix. Instead the minimum-edge adjacency matrix (skeleton matrix) should be altered. All that is required is to replace the 1 corresponding to the edge to be removed with a 0. Since this change affects reachability, it will be desirable for purposes of keeping computer records up to date to construct a new reachability matrix for the structure from which the edge has been removed. This can be done by raising to powers in the manner described in (39–41) of Chapter 9. Further amendments may involve this revised reachability matrix.

EDGE ADDITION. Addition of a new edge to a hierarchy can be done by replacing a 0 in the reachability matrix with a 1, in the position corresponding to the new edge. The revised matrix may not be a reachability matrix; hence it is appropriate to find the transitive closure of the revised matrix and use it thereafter as the reachability matrix of the revised structure. The revised structure can be computed from the revised reachability matrix. The revised structure may no longer be a hierarchy, since addition of an edge may introduce a cycle.

Format Amendments to Hierarchies. Format amendments replace one structure with another. Since format amendments are made to facilitate interpretation, it is to be expected that the replacement structure will be structurally simpler than the original. Two types of format amendment are considered for hierarchies. These are *elementary contraction* and *pooling*. Two selected vertices of a digraph map are involved in each type.

ELEMENTARY CONTRACTION. An elementary contraction is initiated by selecting two adjacent vertices of a digraph map. Suppose the selected pair is (u, v). Then there is an edge directed from u to v on the map. The elementary contraction is accomplished by first deleting from the map the vertex u and the edge from u to v. Next, if there were any vertices adjacent to u that were not also adjacent to v, all such vertices become adjacent to v. The final step is to rename the survivor vertex.

The foregoing graphical description of an elementary contraction is supplemented by a description of the changes to be made in the reachability matrix. The revised reachability matrix will be formed by deleting the row and column indexed by u, replacing the row vector indexed by v with the Boolean sum of the original row vectors indexed by u and v, and making a corresponding change in the columns. Finally the index v is replaced by a new index v' and the transitive closure is formed.

The purpose of contraction can be seen in the context of the Π-Σ process described in Chapter 1. In partitioning a system, the various elements are identified. After a structure has been created, it is possible to carry out integration by contracting the structure. Suppose for example that the vertices u and v represent two elements that can be combined into a single element, such that $v' = u \cup v$. Then the total number of elements being considered is reduced by 1, and the structure is somewhat simpler. Yet the possibility remains to retreat to the more disaggregated original structure if the need to do so arises. Evidently it is desirable, when contracting, to identify a suitable proxy element as a name to replace the names of the two original elements u and v. This permits the new name to be used in discussing the structure.

It may occur that instead of simply contracting two adjacent elements, it is desired to carry out a contraction involving a path of length greater than 1. One effect of multiple elementary contractions can be to replace an entire path involving several vertices with a single vertex. Then there may be no point to naming the replacement vertex anew for each elementary contraction, but it may be adequate simply to identify a new name for the vertex that replaces the whole path. If this is true in some specific instance, the path to be contracted is identified, its elements are collected, its edges are properly amended, and a suitable name for the path proxy is identified. If a path of length k is contracted, evidently the number of elements in the set is reduced by k.

POOLING. Let u and v be two vertices which are not connected, but which lie in the same level or stage of a hierarchy. If these two vertices are replaced by a single proxy element, and the succedents and antecedents of either become the succedents and antecedents, respectively, of the proxy element, the two vertices are said to have been *pooled*.

Pooling diminishes the number of vertexes and edges on a map, and this is its primary value in interpreting a structure. It may or may not be beneficial. Before pooling, the possible disadvantages should be considered along with the presumed advantages.

Substitutions

Substitutions may be made that will have no effect on the reachability matrix. These correspond to substituting a proxy element for a cycle in a multilevel system, for two or more contracted vertices, or for two or more pooled vertices; correcting misspelled words; or rephrasing an element or contextual relation. Rephrasing may also occur as part of a substantive amendment, and hence it is necessary to decide the nature of the amendment before disturbing computer entries.

References

1. J. N. Warfield, *Structuring Complex Systems*, Battelle Monograph No. 4, Battelle Memorial Institute, Columbus, O., 1974.