Neural Networks for Control

Edited by W. Thomas Miller, III, Richard S. Sutton, and Paul J. Werbos

Chapter 5

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5 Adaptive Control Using Neural Networks

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5.1 Introduction

Mathematical systems theory has made major advances in the past four decades and has evolved into a scientific discipline which cuts across boundaries, extending from design and development on the one hand to mathematics on the other. The best developed part of the theory concerns linear systems and important concepts as well as major theoretical results have been introduced in such areas as stability theory, optimal control, multivariable theory and adaptive control. The powerful techniques developed in the area of adaptive control complement current computing technology and have enormous potential in the world of applications, where systems have to be controlled in the presence of uncertainty. Although adaptive systems are by their very nature nonlinear, most of the theories of such systems are deeply rooted in linear systems theory (Narendra and Annaswamy 1989).

In recent years, the multilayer neural network and the recurrent network have emerged as important components, which have proved to be extremely successful in pattern recognition and optimization problems (Hopfield 1982, Hopfield and Tank 1985, Burr 1988, Gorman and Sejnowski 1988, Sejnowski and Rosenberg 1987, Widrow, Winter and Baxter 1988). From the system-theoretic point of view, these networks can be considered as components which can be effectively used in complex nonlinear systems.

The main objective of this chapter is to explore how well-established adaptive identification and control techniques can be applied to the analysis and synthesis of dynamical systems, which contain neural networks as subsystems. It is the author's conviction that this new area will evolve in the near future in the same general directions along which adaptive control did in the past two decades. Further, many of the same theoretical difficulties will also be encountered, though in more complex forms due to the nature of the subsystems involved. In spite of the seeming differences, the concepts developed in adaptive control theory will have great relevance in the new area.

In this chapter, new models for the identification and control of complex nonlinear dynamical systems with unknown parameters are introduced. The assumptions that must be made at every stage to have well posed problems, as well as the theoretical difficulties encountered in assuring stability while using the models for identification and control, are briefly discussed. Finally, areas where further work is needed are suggested.

5.2 Multilayer Networks and Recurrent Networks

Two classes of networks which have received considerable attention in the area of artificial neural networks are (i) multilayer neural networks, and (ii) recurrent networks. A typical multilayer neural network with an input layer, an output layer and two hidden layers is shown in figure 5.1. For convenience, this can be denoted in block diagram form as shown in figure 5.2 with three weight matrices U, V and W. The multilayer network represents a nonlinear map $\hat{f}$ where

$$\hat{f}(x) = F[WF[VF[Ux]]]$$

and the elements of U, V and W are adjustable weights. Such networks have been used successfully in pattern recognition, where the weights are adjusted to minimize a suitable error function. In the following discussions the terms "parameters" and "weights" are used interchangeably.

In contrast to the above, the recurrent network, based on the work of Hopfield (1982), has been used as a content-addressable memory and in optimization problems. One version of the Hopfield network is shown in figure 5.3 and consists of a single layer neural network in the forward path connected to a delay in the feedback path. The choice of the weights determines the equilibrium states of the dynamical systems and hence the specific equilibrium to which the state trajectory converges depends upon the initial conditions. This fact has been used for content-addressable memories as well as optimization problems.

From a system-theoretic point of view, the multilayer network represents merely a versatile nonlinear map. The recurrent network on the other hand, in most cases, represents a dynamical system with no input. In system analysis, both types of operators play an important role. Hence it is desirable to study the properties of feedback systems (refer to figs. 5.5 and 5.6) which contain both types of networks as components. In (Narendra and Parthasarathy 1988b) the backpropagation method used for adjusting the weights in a multilayer neural network was also suggested for adjusting the weights in a recurrent network to increase the number of stored states.

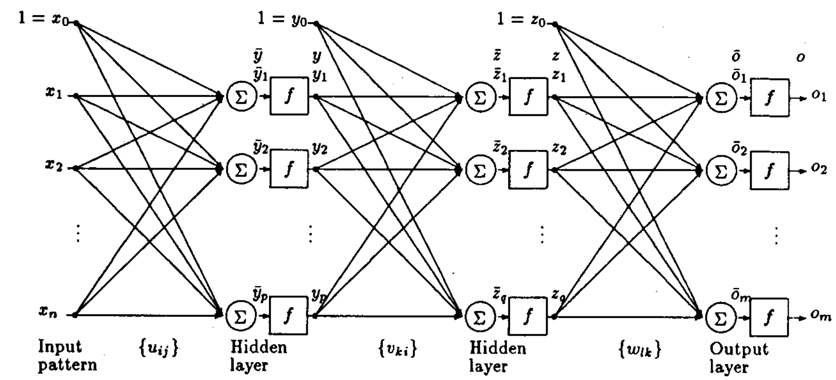

Figure 5.1
A three layer neural network.

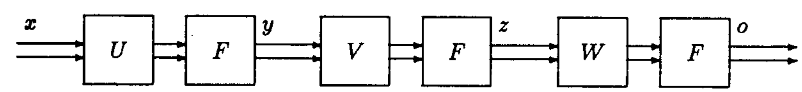

Figure 5.2
Block diagram of a three layer neural network.

5.3 Theoretical versus Empirical Studies

Our main interest is in the control of nonlinear dynamical systems and the networks described in section 5.2 will play an important role in such systems. While one of our primary objectives is to determine theoretical properties such as controllability, observability, and (most important of all) stability of dynamical systems with multilayer neural networks as components, the enormous difficulty of the endeavor is evident right from the outset. Hence, we realize that parallel empirical studies based on computer simulations should form a vital part of any investigation in this general area. In the following sections both aspects of the problems are discussed. Empirical studies will be needed to justify the choice of the

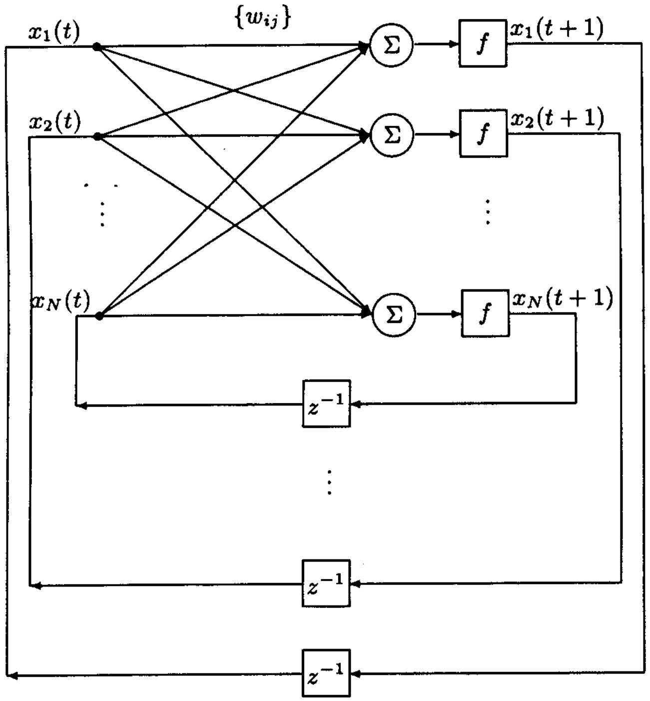

Figure 5.3
The Hopfield network.

problems proposed for further investigation. Theoretical questions which have to be examined in each case are also addressed in each section.

5.4 Multilayer Networks as Static Maps

In the dynamical systems that are considered in the following sections, multilayer neural networks are used as subsystems. For an efficient analysis of the dynamic behavior of the overall system, it is desirable that the mapping properties of the static subsystems be well understood. The following comments concerning the approximating capabilities of neural networks are consequently very relevant for our investigations.

Consider maps in $C(\mathbb{R}^n, \mathbb{R})$ and $C(\mathbb{R}^n, \mathbb{R}^m)$. By Weierstrass's famous theorem, it is known that any function in these classes can be approximated to any degree of accuracy by a polynomial, i.e.,

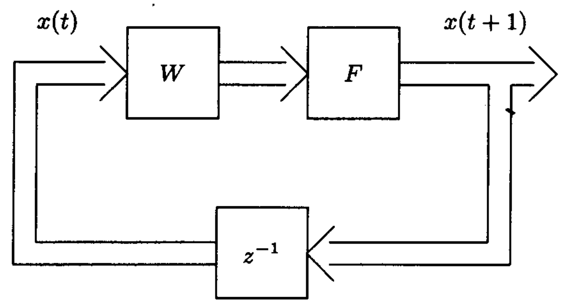

Figure 5.4
Block diagram of a Hopfield network.

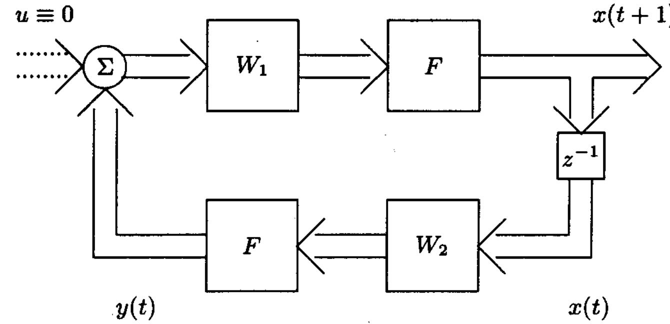

Figure 5.5
Two-layer recurrent network.

$$f(x_1, x_2, \ldots, x_n) \approx \alpha_0 + \sum_i \alpha_i x_i + \sum_i \sum_j \alpha_{ij} x_i x_j + \cdots \tag{5.1}$$

where $\alpha_0, \alpha_i, \alpha_{ij} \in \mathbb{R}$. Similarly, if $\phi_i(x_1, x_2, \cdots, x_n)$ $(i = 1, 2, \cdots)$ are elements of a complete orthonormal set, f can be expressed as

$$f(x_1, x_2, \cdots, x_n) \approx \sum_{i=1}^{N} c_i \phi_i(x_1, x_2, \cdots, x_n). \tag{5.2}$$

In section 5.2, the structure of a three layer neural network was briefly described and the block diagram representation of such a network was given in figure 5.2. More generally, static maps realized by a multilayer neural network with $(n-1)$ hidden layers and an output layer have the form

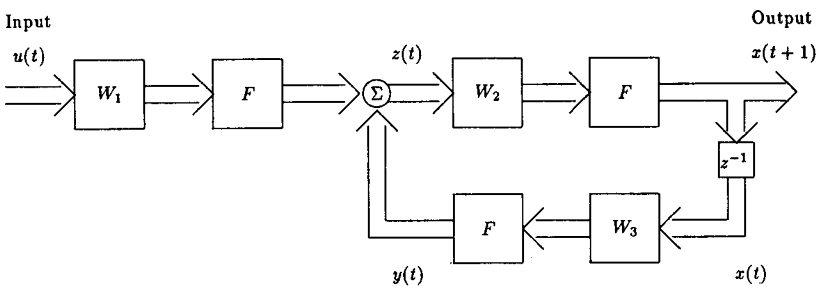

Figure 5.6
Multilayer networks in dynamical systems.

$F[W_1 F[W_2 \cdots F[W_n x]]]$.

The first question that arises is whether such networks can be used to approximate a function $f \in C(\mathbb{R}^n, \mathbb{R})$ or $C(\mathbb{R}^n, \mathbb{R}^m)$ to any desired degree of accuracy.

It has recently been shown in Hornik, Stinchombe, and White (1988), using the Stone-Weierstrass theorem, that a two-layer network with an arbitrarily large number of nodes in the hidden layer can approximate any continuous function $f \in C(\mathbb{R}^n, \mathbb{R}^m)$ over a compact subset of $\mathbb{R}^n$. This implies that neural networks with even one hidden layer are adequate for purposes of characterization.

Since polynomials and orthogonal expansions can also approximate functions in $C(\mathbb{R}^n, \mathbb{R}^m)$ to any degree of accuracy, the advantages of neural networks over such representations are less obvious and have to be justified on the basis of practical considerations. In particular, the following questions have to be addressed if representations using neural networks are to be preferred.

1. Are neural networks more parsimonious representations of special classes of continuous functions in that they need fewer parameters? If so, what are the characteristics of such functions?

2. Given a nonlinear map f which has to be approximated, what dictates the choice of the number of layers and the number of nodes in each layer of the network (i.e. size of the weight matrices W_i)?

These questions, which have received considerable attention, have only partial answers at present. However, such questions are bound to arise with increasing frequency, even as more precise understanding of the behavior of complex dynamical systems is sought in the future. In particular, if neural networks are to be used for the identification and control problems in dynamical systems, their approximation capabilities must be well understood when a finite number of layers with a specified number of nodes in each layer is present in the network. In the absence of such quantitative results, it will be assumed in all the problems that we shall study that the nonlinear function f to be approximated belongs to the class $\mathcal{N}$ of functions that the given neural network can generate for different choices of the weight matrices. This will assure that the identification or control problem is well posed.

5.5 Methods for Adjusting Weights

The adjustment of the weights of a neural network when the latter is used as a component in a dynamical system can be considered as the adaptive part of the control process. In conventional adaptive control theory, the rules for updating the parameter values are called *adaptive laws.* Methods for generating both deterministic and stochastic adaptive laws for controlling linear systems with unknown parameters, based on an output error, are currently well understood. However, when the desired static or dynamic maps are nonlinear, the development of such methods is still in the initial stages.

5.5.1 Backpropagation in Static Systems

Backpropagation is one of the computationally efficient methods for the adjustment of the weights of a neural network. In such a method, the partial derivatives of an error criterion with respect to the weights in a multilayer neural network are determined and the weights in turn are adjusted along the negative gradient to minimize the error function. Figure 5.7 shows a diagrammatic representation of the architecture given in Narendra and Parthasarathy (1988a) for backpropagation and has been extensively used by them in simulation studies involving both static and dynamic systems. The structure of the weight matrices used to compute the derivatives (called the sensitivity network) is seen to be identical to that in the original network, while the signals flow in the opposite direction, justifying the term *backpropagation.* According to the authors, while the diagrammatic representations in figure 5.7 is informationally

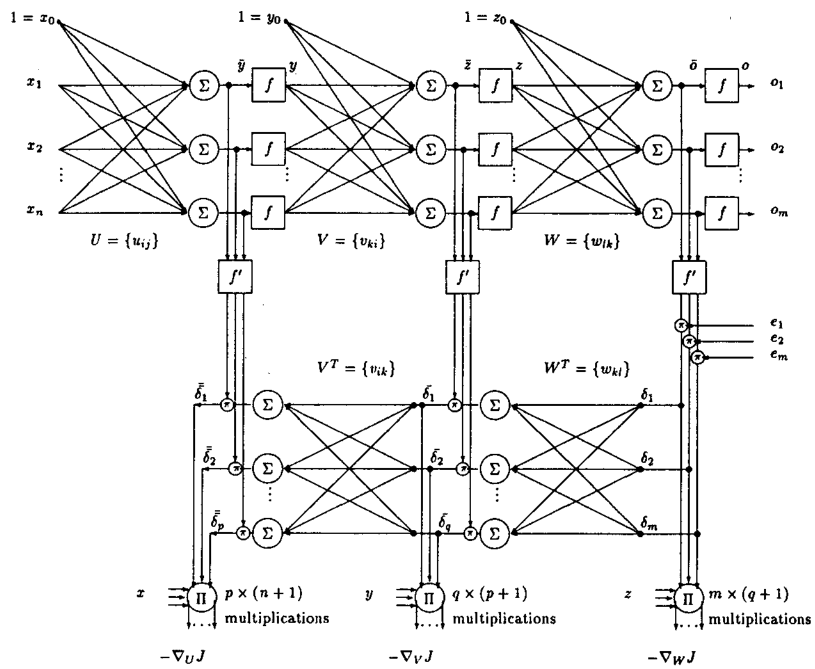

Figure 5.7
Architecture for backpropagation.

equivalent to an algorithmic representation, it enjoys inferential and computational advantages over the latter. These advantages become more evident in the following sections where backpropagation is used in the identification and control of dynamic systems.

5.5.2 Backpropagation in Dynamical Systems

Most physical systems are dynamic in nature and the identification of such systems using input-output data will naturally involve dynamic elements. Further, if such systems are to be controlled efficiently, the powerful concept of feedback has to be used. To have well posed problems, feedback systems must invariably contain dynamic elements (so that algebraic loops are avoided). Hence, if neural networks are to be used for either identification or control, the overall system must include dynamic elements.

If backpropagation is to be the principal method for adjusting the weights of the neural network, the method must be extended to dynamic systems as well. For example, figure 5.8 shows that if time delays are present in cascade in a neural network, the same method shown in figure 5.7 for static neural networks can be suitably modified to yield the desired partial derivatives. The more important question is to determine the partial derivatives when neural networks and linear dynamic elements are connected in arbitrary configurations.

The introduction of dynamics in a system requires a more detailed investigation of the concept of a partial derivative. The reader is referred to Narendra and Parthasarathy (1989b) for a more extensive treatment of this subject. Figure 5.9 shows structures in which neural networks and dynamic elements are connected in cascade and feedback. These represent the basic building blocks of identification and control models. If dynamic backpropagation methods can be developed to determine the partial derivative of the output error with respect to any weight as a function of time, gradient methods can be extended to dynamic systems also. It is precisely this approach that is used in sections 5.6 and 5.8.

Given a dynamical system with a multilayer network as a component and an output error $e(k)$ at time k, the backpropagation method yields $\frac{\partial e(k)}{\partial w_{ij}}$ as the output of a dynamical sensitivity network, where w_{ij} is a typical weight. It is assumed that w_{ij} is a constant so that a partial derivative can be rigorously defined. This implies that changes in w_{ij} should be small and should be made over large time intervals. When w_{ij} is adjusted at every instant and the step size is large, it is no longer possible to theoretically justify the adjustment of the weights of the network. However, local stability as well as satisfactory performance can be achieved by making the parameter adjustments sufficiently small.

5.5.3 Stable Adaptive Laws

It is of historical interest to note that in the early 1960s, gradient methods were used widely in the identification and control of linear dynamical systems (Narendra and McBride 1964, Kokotovic 1964, Cruz 1973). The performance of such systems, demonstrated through simulations, was for the most part satisfactory. However it was shown by Parks (1966) that systems in which parameters are adjusted in such a fashion can lead to instability. It was this inability to prove stability of rapidly adjusting systems using gradient methods that eventually led to their demise and

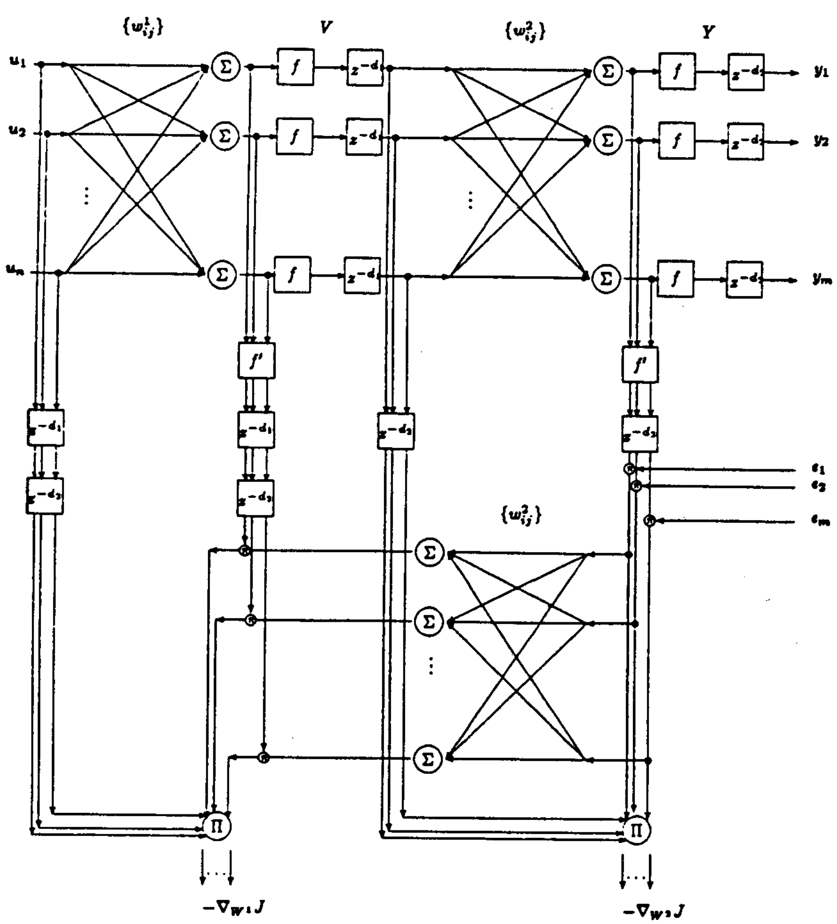

Figure 5.8
Architecture for backpropagation when delays are present.

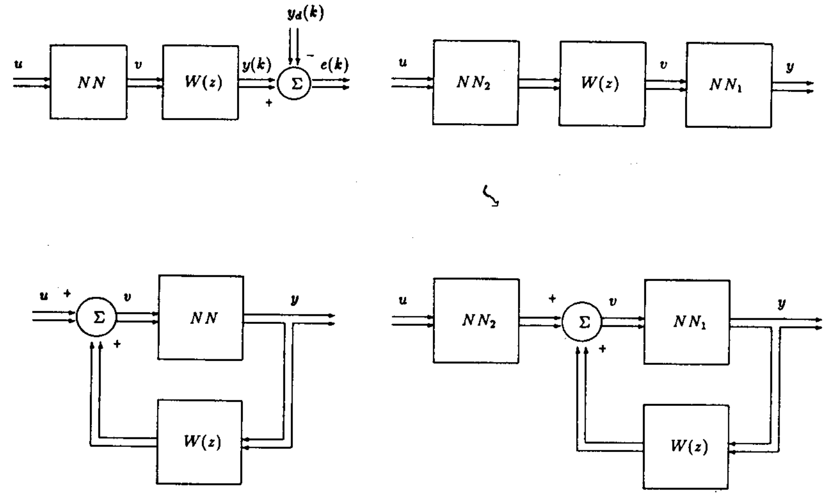

Figure 5.9
Building blocks of identification and control models.

witnessed the rise of stability methods that are currently used in the field.

A large literature currently exists on the stability of adaptive systems where the process to be controlled is linear and time-invariant. The adaptive procedure makes the overall system nonlinear in such cases. Liapunov's direct method and hyperstability theory have been applied extensively to prove the stability of such systems. The concept of positive realness is intimately related to stability and hence to the generation of stable adaptive laws. The Kalman-Yacubovich lemma is an important link in relating the positive realness of a transfer function and the existence of a Liapunov function.

The problem of stability is vastly more difficult when artificial neural networks are used either for identification or control and the system is nonlinear. Unlike linear systems, simple algebraic conditions are not available for assuring the stability of the overall system. Even if stability is assumed, a corresponding Liapunov function cannot be easily determined. Further, Liapunov stability (uniform asymptotic stability) of the equilibrium state does not assure bounded-input bounded-output stability. The concept of positive realness has to be replaced by the concept of passivity to prove stability. All these indicate that our knowledge of the stability of dynamical adaptive systems using artificial neural networks is quite rudimentary at the present time and that considerable work remains to be done. A large body of work exists in adaptive control theory in which the adaptive systems, linearized around nominal values, are analyzed. These methods will also find application in the design of stable adaptive controllers using artificial neural networks. However, in the author's opinion, such methods based on linearization will be of limited use in problems where the initial weight vector in the neural network is far from the desired value. It is precisely in problems where the system has to adapt to large uncertainty that controllers based on neural networks will be needed in practical applications. For such problems, new concepts and methods based on stability theory will have to be explored. Until such methods are developed, we have to resort to dynamic backpropagation methods to evaluate different architectures for identification and control models.

5.6 Models for the Identification of Nonlinear Dynamical Systems

In this section four models which were introduced in Narendra and Parthasarathy (1989a) for the representation of a single-input single-output (SISO) nonlinear plant are presented. These models were chosen both for their generality as well as analytical tractability. The models are motivated by corresponding models which have been used in the adaptive systems literature for the identification of linear systems and can be considered as their generalizations to nonlinear systems. Since backpropagation is the principal method that we shall use for the adjustment of parameters of the identification model, the parameterization of the plant (and hence the model) is such as to make the application of the procedure relatively straightforward.

5.6.1 Characterization of Plants

The models of the four classes of plants introduced here can be described by the following nonlinear difference equations:

$$\begin{aligned}
\text{Model I:} \quad x(k+1) &= \sum_{i=0}^{n-1} \alpha_i x(k-i) + f[u(k), u(k-1), \\
&\quad \ldots, u(k-m+1)] \\
\text{Model II:} \quad x(k+1) &= f[x(k), x(k-1), \cdots, x(k-n+1)] \\
&\quad + \sum_{i=0}^{m-1} \beta_i u(k-i) \\
\text{Model III:} \quad x(k+1) &= f[x(k), x(k-1), \cdots, x(k-n+1)] \\
&\quad + g[u(k), u(k-1), \cdots, u(k-m+1)] \\
\text{Model IV:} \quad x(k+1) &= f[x(k), x(k-1), \cdots, x(k-n+1); \\
&\quad u(k), u(k-1), \cdots, u(k-m+1)]
\end{aligned}$$

where $[u(k), x(k)]$ represents the input-output pair of the SISO plant at time k. The functions f and g are assumed to be differentiable functions of their arguments. It is evident that Model IV subsumes Models I-III. However, Model IV is analytically the least tractable and hence for practical applications, some of the other models might prove more attractive. In this section, each of these models is briefly described.

Model I: The output of the unknown nonlinear plant in this case is assumed to depend linearly on its past values and nonlinearly on the past values of the input. The latter is realized as shown in figure 5.10 and consists of tapped delay lines at the input and the feedback path.

Model II: This model is realized as shown in figure 5.11. In this case, the output depends linearly on the input $u(k)$ and its past values but nonlinearly on its own past values. The advantage of this model is that it lends itself readily to control in practical situation as shown in section 5.8.

Model III: The unknown plant in this case is described by a nonlinear difference equation of the form

$$\begin{aligned}
x(k+1) &= f[x(k), x(k-1), \cdots, x(k-n+1)] \\
&\quad + g[u(k), u(k-1), \cdots, u(k-m+1)]
\end{aligned} \tag{5.3}$$

and hence depends nonlinearly on both its past values as well as those of the input. However, the effects of the input and output values are additive as shown in equation (5.3). The representation of equation (5.3) is shown in figure 5.12.

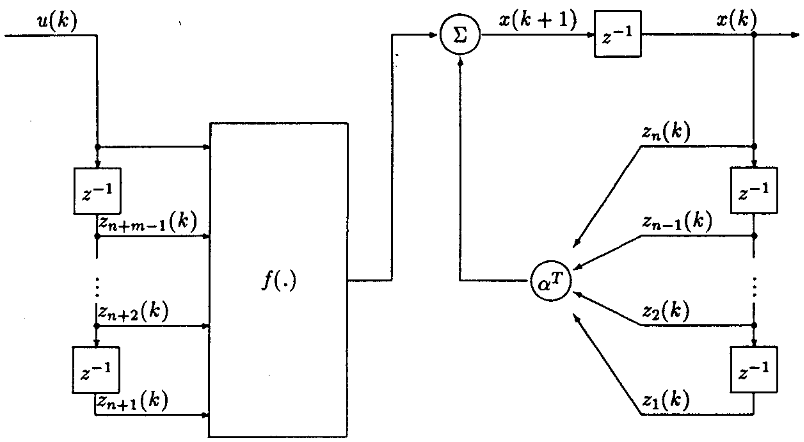

Figure 5.10
Model I: Structure of the plant.

Model IV: As mentioned earlier, this is the most general of all models introduced here and subsumes the earlier models. The output at any instant in this case is a nonlinear function of the past values of both the input and the output. Once again, the representation of the model using tapped delay lines is shown in figure 5.13.

5.6.2 Identification Procedure

The identification model of the plant is composed of neural networks and tapped delay lines. In each case, the neural network is assumed to contain sufficient number of layers, and nodes in each layer, so as to be able to match exactly the input-output characteristics of the corresponding nonlinear mapping in the given plant. From the point of view of mathematical analysis, this implies that the nonlinear functions in the difference equations describing the plant can be replaced by neural networks with fixed but unknown weight matrices W_i^*. Hence, a theoretical solution to the adaptive identification problem is assumed to exist from the outset.

To identify the plant, an identification model is chosen based on prior information concerning the class to which it belongs. For example, assuming that the plant has a structure described by Model III, the model

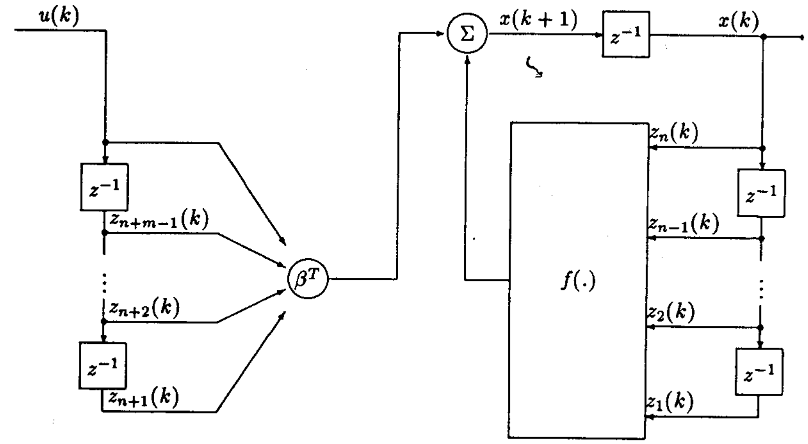

Figure 5.11
Model II: Structure of the plant.

is chosen to have the form shown in figure 5.14. The aim then is to determine the weights of the two neural networks N_1 and N_2 so that the mapping N_1 is equal to $g[.]$ and the mapping N_2 is equal to $f[.]$. If $x(k+1)$ and $\hat{x}(k+1)$ are respectively the outputs at stage $k+1$ of the plant and the identification model, the error $e(k+1) \triangleq \hat{x}(k+1)-x(k+1)$ is used to update the weights of N_1 and N_2. As described in Section 5.5, either static or dynamic backpropagation is used, depending on the structure of the identifier used.

a. Parallel Model: In this case, the structure of the identifier is identical to that of the plant with f and g replaced by N_2 and N_1 respectively. This is shown in Figure 5.14. Since N_2 is in a dynamic feedback loop, the parameters of N_1 and N_2 have to be adjusted using dynamic backpropagation.

b. Series-Parallel Model: In this case $x(k+1)$ rather than $\hat{x}(k+1)$ is used to generate the output of the model. This implies that the model is described by the equation

$$\hat{x}(k+1) = N_2[x(k), \cdots, x(k-n+1)] + N_1[u(k), \cdots, u(k-m+1)].$$

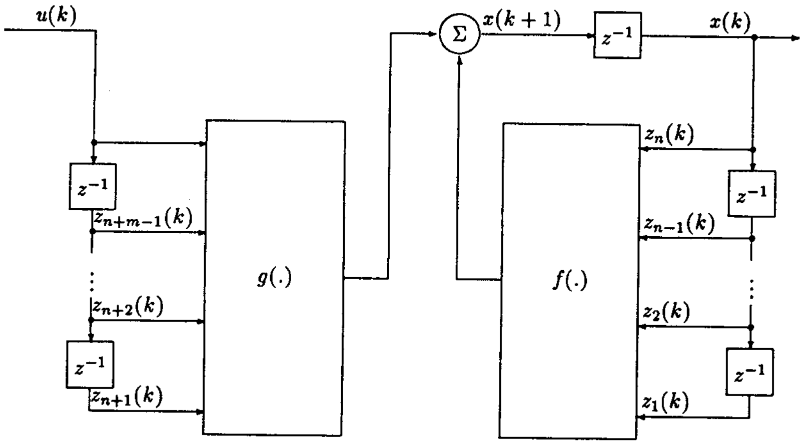

Figure 5.12
Model III: Structure of the plant.

Since the model does not include a feedback loop containing a nonlinear element, static backpropagation rather than dynamic backpropagation of the error can be used to adjust the weights of the neural network.

The two methods outlined above have been discussed extensively in the context of the identification of linear time-invariant systems with unknown parameters (Narendra and Annaswamy 1989). While the series-parallel method has been shown to be globally stable, similar results are not available for the parallel model. To avoid many of the analytical difficulties encountered, as well as to assure stability and simplify the identification procedure, the series-parallel model was used in Narendra and Parthasarathy (1989a). Extensive computer simulations have revealed that a large class of nonlinear plants can be identified using the above procedure. However, theoretical studies concerning stability and convergence are still in the initial stages and numerous questions have yet to be answered.

Comment 1: The identification of an unknown nonlinear plant may be needed for off-line or on-line control. The nature of the identification in the two cases may be substantially different. The various considerations that determine the models to be used and the test signals to be applied need to be studied further.

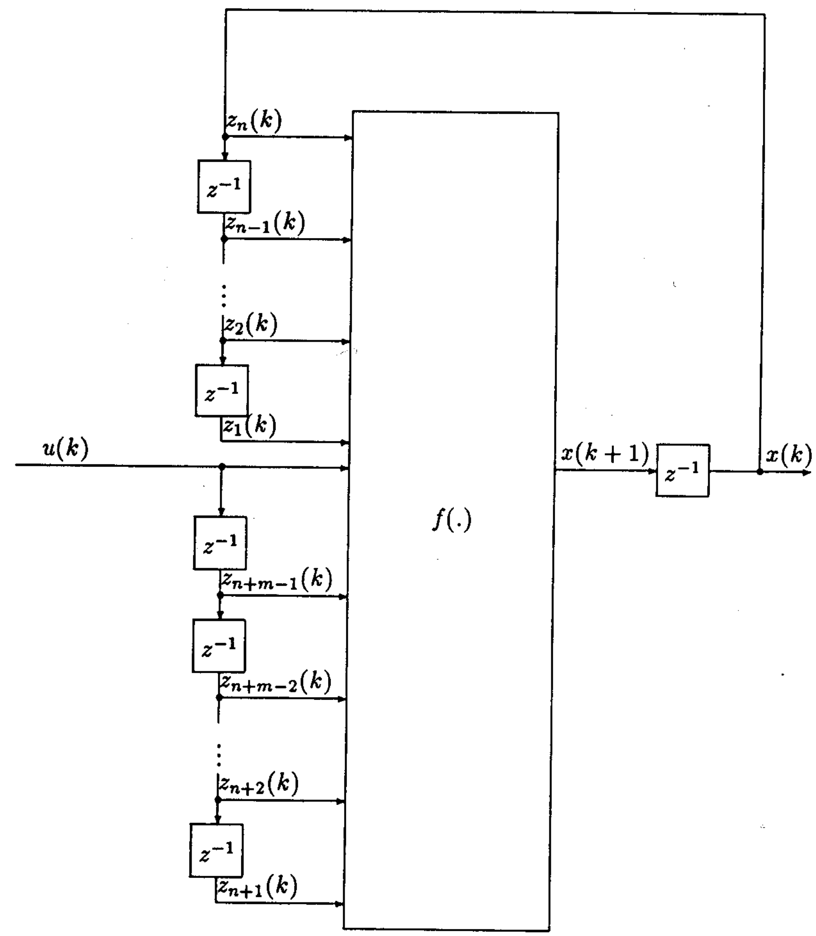

Figure 5.13
Model IV: Structure of the plant.

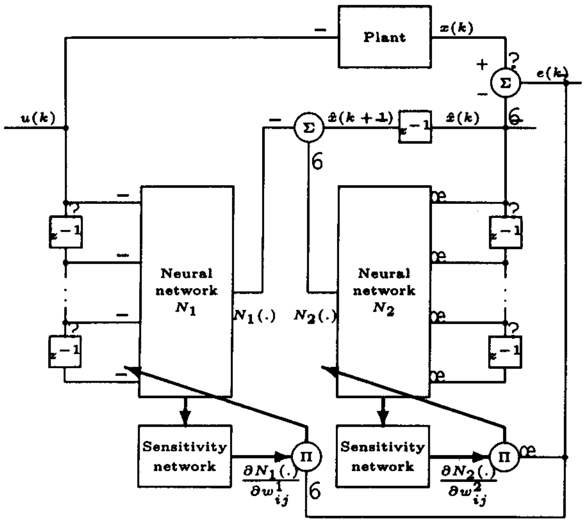

Figure 5.14
Model III: Structure of identification model.

Comment 2: The back-propagation method yields the partial derivatives of an error function with respect to a set of parameters at every instant of time. However, in practice, the parameters are continuously varied, violating the assumptions made in deriving partial derivatives. The additional assumptions that have to be made concerning the rate at which the parameters are adjusted to assure overall system stability need further investigation.

Comment 3: In all the simulation studies, the output error tends to zero rapidly but the convergence of the weights cannot be assured. Concepts similar to persistent excitation encountered in adaptive systems theory are needed in this case also to assure convergence. This is briefly discussed in the following section.

5.7 Persistent Excitation

The importance of the class of inputs to be used to train adaptive and learning systems is generally acknowledged. The training set has to be representative of the entire class of inputs that the system may be subjected to. This will ensure that the system will respond in the desired fashion even when an input not included in the training set is applied to it. This concept, referred to as persistent excitation, has been extensively treated in conventional adaptive control theory both in the context of identification and control problems.

Let θ be an adjustable parameter vector in a system and let θ^* be a constant vector such that when $\theta \equiv \theta^*$, the system has the same transfer function as the model. The vector θ^* may be unique or belong to a set $\mathcal{S}$. In the latter case, the given system has the same input-output behavior as the given model for all constant values of $\theta \in \mathcal{S}$. If specific inputs are applied to the system as well as the reference model, the output error may tend to zero asymptotically with time even if $\theta \notin \mathcal{S}$. However, if the input is persistently exciting, $\theta(k) \rightarrow \theta^* \in \mathcal{S}$. If θ^* is unique, this also implies that the parameters of the unknown system can be identified exactly. Conditions that ensure the persistent excitation of the reference input which will result in parameter convergence are well established in the adaptive control literature.

The concept of persistent excitation is also found to be important while dealing with the identification and control of nonlinear systems using neural networks. Let $\mathcal{U}$ represent the compact set to which the input u belongs. If a specific input $u \in \mathcal{U}$ is chosen (e.g., a single sinusoid), the output error rapidly approaches zero if back-propagation is used to adjust the weights of the neural network. However, if $\mathcal{L}$ and $\hat{\mathcal{L}}$ represent the operators of the plant and the identification model respectively, $\hat{\mathcal{L}}$ is not "close" to $\mathcal{L}$ in any sense i.e. the norm $||\mathcal{L}u - \hat{\mathcal{L}}u||$ is not small for all $u \in \mathcal{U}$. This becomes evident by considering an input u not included in the training set.

If the input to the plant (and the identification model) is sufficiently general and the weights of the neural network are adjusted for a sufficiently long time, the output error can be made small for any input in $\mathcal{U}$. The general input referred to above may then be considered to be persistently exciting. Needless to say, methods of characterizing persistently exciting inputs, which assure the convergence of the identification model to the desired set, are highly desirable.

If the operator $\hat{\mathcal{L}}$ approximates $\mathcal{L}$ in some sense, the model can be used off-line in the place of the plant for prediction purposes. However if accurate model following is achieved only by adjusting the parameters of the model at every instant, the latter has limited predictive ability, but can be used effectively on-line. It is this method that is used in the following section for the adaptive control of an unknown plant.

5.8 Adaptive Control

Parametric adaptive control is the problem of controlling the output of a system with a known structure but unknown parameters. To make the problem analytically tractable, in adaptive systems theory the plant to be controlled is assumed to be linear time-invariant with unknown parameters. These parameters can be considered as the elements of a vector p. If p is known, the parameter vector θ of a controller can be chosen as θ^* so that the plant together with the fixed controller behaves like a reference model described by a linear difference (or differential) equation with constant coefficients. If p is unknown, the vector $\theta(t)$ has to be adjusted on-line using all the available information concerning the system.

Two distinct approaches to the adaptive control of an unknown plant are (i) direct control and (ii) indirect control. In direct control, the parameters of the controller are directly adjusted to reduce some norm of the output error. In indirect control, the parameters of the plant are estimated as $\hat{p}(t)$ at any time instant and the parameter vector $\theta(t)$ of the controller is chosen assuming that $\hat{p}(t)$ represents the true value of the plant parameter vector. Even when the plant is assumed to be linear and time-invariant, both direct and indirect adaptive control result in nonlinear systems. When the plant is nonlinear and dynamic (i.e. the present value of its output depends upon the past values of the input and the output respectively), a neural network can be used as a controller as shown in figure 5.15. This corresponds to direct control.

Direct Control: In conventional direct adaptive control theory, methods for adjusting the parameters of a controller based on the measured output error rely on concepts such as positive realness and/or passivity. By making suitable assumptions concerning the plant and the reference model, it is shown that the direction in which a parameter is to be adjusted can be obtained by correlating two signals that can be measured. Using either Liapunov theory or hyperstability theory it is shown that the adjustments of all the controller parameters based on such adaptive laws result in the stability of the overall system.

At present, methods for directly adjusting the parameters of the controller (the neural network N_c in figure 5.15) in a stable fashion based on the output error are not available. This is due to the nonlinear nature of both the plant and the controller. Even backpropagation cannot be used directly, since the plant is unknown and hence cannot be used to generate the desired partial derivatives. Hence, until direct control

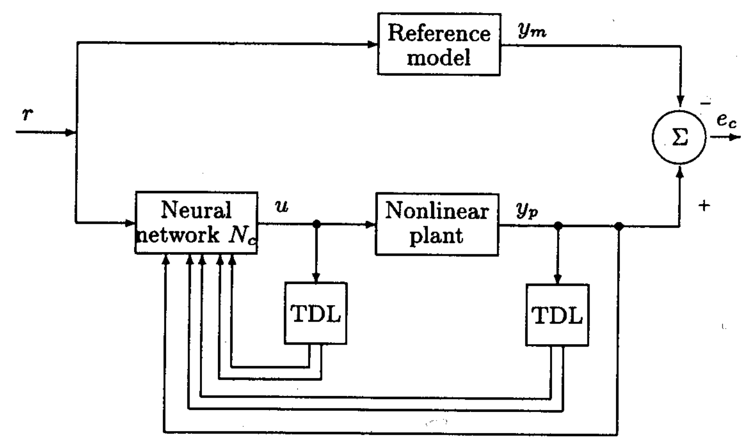

Figure 5.15
Direct adaptive control using neural networks.

methods are developed, adaptive control of nonlinear dynamical systems has to be carried out using indirect control methods.

Indirect Control: As mentioned earlier, when indirect control is used to control a nonlinear system, the plant is parameterized using one of the models described in the previous section and the parameters of the model are updated using the identification error. The controller parameters in turn are adjusted by backpropagating the error (between the identified model and the reference model outputs) through the identified model. A block diagram of such an adaptive system is shown in figure 5.16.

Both identification and control can be carried out at every instant or after processing the data over finite intervals. When external disturbances and/or noise are not present in the system, it is reasonable to adjust the control and identification parameters synchronously. However, when sensor noise or external disturbances are present, identification is carried out at every instant while control parameter updating is carried out over a slower time scale, to assure robustness (i.e., control parameters are adjusted with a lower frequency than identification parameters).

Prior Information: In conventional adaptive control theory, where the plant is linear and time-invariant but with unknown parameters, stable adaptive laws have been derived only by assuming that considerable prior information concerning the plant transfer function is available. In

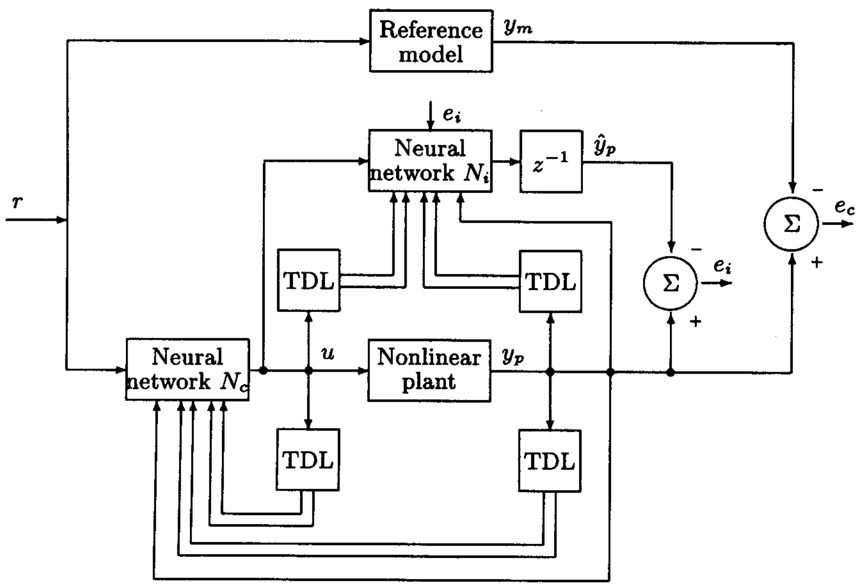

Figure 5.16
Indirect adaptive control using neural networks.

particular it is assumed that (i) the sign of the high frequency gain, (ii) the order of the plant, and (iii) the relative degree of the plant transfer function are known, and that (iv) the zeros of the plant lie in the interior of the unit circle. It is only reasonable to assume that considerably more prior information concerning the plant input-output characteristics has to be assumed when the plant is nonlinear, before theoretical results can be derived to control it in a stable fashion. The insights developed in controlling linear systems are bound to prove invaluable in this context. For example, if the zeros of the plant transfer function lie outside the unit circle it is possible for the plant input to grow in an unbounded fashion even as the output error tends to zero. Assumptions ii and iii are made to assure the existence of the desired controller as well as the reference model. Theoretical studies on the control of nonlinear dynamical systems with unknown parameters will have to address similar questions in the future.

Adaptive Control of Nonlinear Dynamical Systems: The overall structure of the adaptive system proposed in this paper to control a nonlinear dynamic system is the same as that shown in figure 5.16 and

is independent of the specific model used to identify the plant. The latter depends critically on the prior information that is available. The delayed values of the plant input and plant output form the inputs to the neural network N_c which generates the feedback control signal to the plant. The parameters of the neural network N_i are adjusted by backpropagating the identification error e_i while those of the neural network N_c are adjusted by backpropagating the control error (between the output of the reference model and the identification model) through the identification model.

Assuming that one of the models suggested in section 5.6 is used to identify the plant, the problem of determining a control input which will result in the output error tending to zero asymptotically can still be a formidable one. This is because general methods do not exist even for controlling nonlinear plants whose input-output characteristics are completely known. Of the four identification models suggested earlier, only Model II lends itself readily for control purposes. For example, let the plant be described by the difference equation

$$x(k+1) = f[x(k), x(k-1), \cdots, x(k-n+1)] + u(k)$$

and the reference model by

$$x_m(k+1) = \sum_{i=0}^{n-1} \alpha_{mi} x_m(k-i) + \sum_{j=0}^{m-1} \beta_{mj} r(k-j)$$

where $\alpha_{mi}, \beta_{mj} \in \mathbb{R}$, and the poles and zeros of the model transfer function lie in the interior of the unit circle. Let the estimate of f at stage k, based on the inputs and outputs observed upto time instant $k-1$ be denoted by $\hat{f}_k$. $\hat{f}_k$ is determined by updating the weights of a neural network by using the identification error

$$\begin{aligned} e_i(k) &= x(k) - \hat{x}(k) \\ &= f[x(k-1), x(k-2), \cdots, x(k-n)] \\ &\quad - \hat{f}_{k-1}[x(k-1), x(k-2), \cdots, x(k-n)] \end{aligned}$$

in the backpropagation method. The control input $u(k)$ is then updated as

$$\begin{aligned} u(k) &= -\hat{f}_k[x(k), x(k-1), \cdots, x(k-n+1)] + \sum_{i=0}^{n-1} \alpha_{mi} x_m(k-i) \\ &\quad + \sum_{j=0}^{m-1} \beta_{mj} r(k-j). \end{aligned} \tag{5.4}$$

The analytical problem is then to show that the output $x(k)$ of the plant tends asymptotically to that of the reference model. If Model I is used to represent the plant so that

$$x(k+1) = f[x(k), x(k-1), \cdots, x(k-n+1)] + g[u(k)]$$

a similar procedure can be adopted if the right inverse of g (denoted by $g^{\dagger}$) exists such that $gg^{\dagger}u(k) = u(k)$ over the range of interest. In such a case, $g^{\dagger}$ has to be estimated on-line for use in the controller. Models III and IV where the equations describing the plant have the form

$$x(k+1) = f[x(k), \cdots, x(k-n+1)] + g[u(k), \cdots, u(k-m+1)]$$

and

$$x(k+1) = f[x(k), \cdots, x(k-n+1); u(k), \cdots, u(k-m+1)]$$

are considerably more difficult to control theoretically. The simulation studies merely serve to provide empirical evidence of the effectiveness of neural networks as controllers in those situations where no reliable theory exists.

The Reference Model: In the control problem discussed above, it is tacitly assumed that the reference model is linear. This is because linear systems theory is well developed and methods of choosing linear models that have desired properties are well established. In contrast to this, methods of determining general nonlinear models with desired transient and steady state properties are not currently available. This accounts for the use of linear reference models in may practical applications.

Whether the reference model is linear or nonlinear, an important practical consideration is that the dynamic mapping represented by the system, containing the controller and the nonlinear plant, should approximate that of the reference model as $k \to \infty$. Hence, an important theoretical question concerns the generation of reference models which satisfy this criterion.

Neural Networks as Adaptive Controllers: A fundamental question that has to be answered is where neural networks would prove most effective in adaptive control problems. Since the adaptive control of linear systems is highly developed at present, the principal applications of neural networks are bound to be in the control of nonlinear systems. Four distinct classes of problems where such nonlinear control may be desirable are listed below. In each case the plant is assumed to be described by a complex nonlinear difference (or differential) equation.

i. The first class includes plants whose governing equations are completely specified. Hence a controller can at least be designed in theory. However, since the determination of such a controller may be too complex for practical purposes, it may be preferable to use a neural network in an adaptive fashion.

ii. In some situations (e.g., flexible space structures) the plant may be stable but have a poor transient response for initial conditions in a domain of interest. Further a linear feedback controller may not be satisfactory for large initial conditions. In this case, identification of the plant using a neural network over a long period of time followed by the use of a nonlinear controller may result in a better overall response.

iii. In some cases (e.g., chemical processes) the plant may have to operate efficiently at several equilibrium states. The purpose of an adaptive controller in such a case is to stabilize the system around one of these equilibrium states depending upon the information available at any moment. While very little theoretical work has been reported in this area, simulation results indicate that controllers based on neural networks may prove effective in such contexts.

iv. In many situations, control may be carried out on the basis of output patterns. The state space in such a case is partioned into disjoint regions which are equivalent for purposes of control. Neural networks may be used to generate the optimal input corresponding to each region.

Work on most of the problems described above is still in the initial stages.

5.9 Conclusion

In this chapter the use of neural networks for the identification and control of nonlinear dynamical systems is proposed. Many of the concepts and methods developed in adaptive control theory, for controlling linear systems with unknown parameters, are shown to have their counterparts in this problem. Models of nonlinear plants are first suggested whose structures are motivated by models which have proved successful in the identification of linear time-invariant plants. Using indirect control, the identified plant models are used to determine the weights of the controller. Both in identification and control problems, backpropagation (both static and dynamic) is used to adjust the relevant weights.

It is shown throughout the paper that to have a well-developed theory of control using neural networks, a vast number of questions have to be first resolved. Many of these are closely related to the stability of dynamical systems in which neural networks are used as subsystems. One of the major problems that is worth investigating and which appears to be tractable is the generation of stable adaptive laws for adjusting the weights of the neural networks using signals that are readily available. The extensive simulations carried out using the models suggested in this paper reveal that neural networks are very effective both for identification and control. This provides adequate justification for current efforts to address the truly formidable theoretical questions that have been generated.

Acknowledgments

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